Parameter Estimation of Multi-component Chirp Signal Based on Differential Evolution

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Abstract: Chirp signals show energy aggregation in the fractional Fourier domain (FrFD) which can be used to estimate the parameter of the signals. In this paper, a parameter estimation method for multi-component chirp signal which corrupted by white Gaussian noise is proposed based on the discrete fractional Fourier transform (DFrFT) and the differential evolution (DE) algorithm. The proposed algorithm uses the DE algorithm instead of the conventional fine search algorithm to detect the peak of the signals in the FrFD. The paper simulated the influence of the noise and the resolution of the proposed algorithm. The results of the simulation show the proposed method does not only improve the estimation accuracy of the peak coordinate, but also reduces time consuming.

Keywords: Chirp Signal; Fractional Fourier Transform; Parameter Estimation; Differential Evolution

1 Introduction

Linear frequency modulated (LFM or chirp) signals have been widely applied in various areas of science and engineering[1-7]. Chirp signal is a typical bandlimited signal in the fractional Fourier domain (FrFD), which exhibits a linear change in instantaneous frequency with time. Chirp signals are of particular interest for sonar systems, radar communications and channel characterization [8, 9]. The frequency modulated rate of chirp signals can be interpreted as a convergence order in the FrFD. It is important to find out the fractional order for detecting or analysis of bandlimited signal in the FrFD. In this paper, we are interested in this problem.

The parameter estimation is of interest in numerous engineering fields, such as radar identification[1]. Numerous chirp-rate estimated algorithms which have been suggested in the literature are the excellent solutions to this task. The maximum likelihood (ML) estimation is a good method to estimate parameters of chirp signal. Although the maximum likelihood ratio test has been proved to be optimal for chirp detection, the large computational complexity it required makes it difficult to apply to practical problems[1,2].

In recent years, many techniques based on different theories have been proposed. The authors[7, 10] proposed a method based on short-time FrFT and wavelet transform, which is suitable for analysis of multi-component and nonlinear chirp signals. However, the resolution performance suffers from the selection of window functions and the width of windows. A method based on Wigner-Ville transform (WVT) is proposed[1]. When dealing with a single chirp signal, the method can highlight the instantaneous frequency and process such signals perfectly. However, the WVT method creates cross terms which bring interference to the results when it deals with the multi-component chirp signal. So this method cannot complete the multi-component chirp signal processing. Even though the improved WVT algorithm[5] offers a good rejection capability of the cross terms, it sacrifices the accuracy of the parameter estimation. The authors[11] propose the generalized S-transform which is a linear invertible transform and has no cross term. However, the performance of time-frequency localization will be inevitably degraded so that the estimation performance of the weak components of Chirp signals will be interfered. Jin *et* *al.*[12] proposed a method based on ambiguity function which can suppress the interference created by cross term. But through the theorem we can see it is difficult for the computer program to obtain the precise solutions of multi-component chirp signals. Liu *et al.* [13] proposed a novel algorithm that combines segmented discrete polynomial-phase transform and sparse discrete fractional Fourier transform to yield a significant reduction of the computational load with a satisfactory estimation performance.

There are also some other methods which combine the time-frequency methods with other algorithms. The authors[14] proposed a method to separate the signal component using Pseudo Wigner-Ville Distribution (PWVD) combined with Short-time Fourier Transform (STFT). The key step of the whole algorithm is to find the four order central moment of FrFT, which brings large amount of calculation. A method based on FrFT and narrow band filter is put forward which is effective in detection and parameter estimation of multi-component Chirp signal [15]. The method uses narrow band filter to isolate the weak signals from the origin chirp signal. The influence caused by strong signals to the weak signals can be suppressed. However, the width of the chosen narrow band filter has deep influence on the result of the parameter estimation. The process of filtering inevitably results in the lost of the weak signal energy because the window function removes all the energy in the stop band. The paper[16] shows us a efficient arithmetic based on direct and spline interpolation. Compared to the conventional searching algorithm, the method does not only improve the estimation accuracy of the single chirp signal's peak coordinate but also save computation load. The algorithm based on Quasi-Newton method[18] has the similar advantages on computation complexity and accuracy for the detection of single chirp signal. However, these algorithms mentioned[18, 19] exist shortcomings when dealing with multi-component chirp signals. They only find one global optimal solution (the strongest signal component) and cannot obtain the local optimal solutions (weak signal components) at the same time.

In this paper, a practical estimation method is proposed, which combines the differential evolution (DE) algorithm and fractional Fourier transform. Since the FrFT is a 1-D linear transform and can be computed by FFT, this method is more efficient in computation. Combining with the DE algorithm, multiple optimal solutions can be obtained at one time. The performance of parameter estimation and the efficiency of computation will be greatly improved.

The remainder of this paper is organized as follows. Section II introduces some preliminaries, such as the signal model of multi-component chirp signals, definition of the discrete FrFT and differential evolution. In section III, the idea and procedure of the multi-component method is analyzed and the implementation steps are illustrated. The parameter estimation process of multi-component chirp signal consists of rough search and high precision search. In section IV, we verify the performance of the proposed methods by numerical simulation, and analysis the results of the simulation. Finally, concludes the paper in section V.

2 Preliminaries and problem formulation

2.1 Fractional Fourier Transform

The fractional Fourier transform (FrFT) is a generalized version of the conventional continuous Fourier transform. In contrast to the standard Fourier transformation, the FrFT has gained considerable attention in Fourier optics, quantum mechanics, radar, variant filtering, signal processing[17], because the FrFT makes a connection between time domain and frequency domain with an additional degree of freedom. Essentially, the representation of a signal in the FrFD contains the information of the signal in both time and frequency domains. The definition of the FrFT is denoted by follows[18]:

$$X_\alpha(u) = F^\alpha\{x(t)\} = \int_{-\infty}^{+\infty} K_\alpha(t,u)x(t)\,\mathrm{d}t \tag{1}$$

where F^{α} denotes the FrFT operator. The kernel function $K_{\alpha}(t,u)$ is given by:

$$K_{\alpha}(t,u) = \begin{cases} A_{\alpha} e^{j\pi(t^2\cot\alpha + u^2\cot\alpha - 2tu\csc\alpha)}, & \alpha \neq k\pi \\ \delta(u-t), & \alpha = 2k\pi \\ \delta(u+t), & \alpha = (2k+1)\pi, \end{cases} \quad (2)$$

where $A_{\alpha} = \sqrt{1 - j\cot\alpha}$ and $k \in \mathbb{Z}$. Here we use the computational method proposed by Majorkowska-Mech *et al.*[23]. This method decomposed the original DFFT matrix as an algebraic sum of a dense matrix and of one or two another matrices which have many zero entries. Thus the calculation requires a small number of arithmetic operations, and the DFrFT computation is derived based on the matrix factorization.

2.2 Differential Evolution

Differential evolution (DE) [20, 21] is an evolutionary method which is used to solve continuous optimization problem by swarm intelligence. DE optimizes a problem by iteratively improving a candidate solution with regard to a given measure of quality. DE is used for multi-dimensional real-valued functions which does not require the optimization problem to be differentiable. DE has advantages such as ease of implementation, the nature of parallelism and good global search ability and so on.

The DE algorithm works by choosing a population of candidate solutions. The candidate solutions are chosen in the search-space by combining simple mathematical formulae and the positions of existing solutions. The new solution will be kept if there is an improvement, otherwise the new solution is simply discarded. The process is repeated and the optimal solution is hoped, but not guaranteed, that a satisfactory solution will eventually be discovered. The process of the DE is similar with other evolutionary algorithm including initialization, mutation, crossover and selection. The essence of the DE is the differential policy by which the individual realized update to be a new variability. DE improves the ability of adaptive search by efficient use of distribution groups. The main process is shown in Fig. 1.

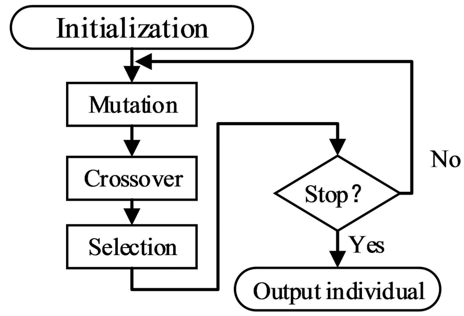


Fig. 1 The steps of DE algorithm

2.3 Problem Formulation

The problem can be defined as finding the fractional order α which makes the bandwidth of signal as narrow as possible. That means the energy of signal would aggregate in a small bandwidth. Correspondingly there would be a peak in the band of signal. There is a parametric model for the phase of the signals components. The model of the chirp signal is denoted by following:

$$\begin{aligned} x_i(t) &= a_i \mathrm{rect}\left(\frac{t}{T_i}\right) e^{\Theta_i(t)} \\ &= a_i \mathrm{rect}\left(\frac{t}{T_i}\right) e^{j2\pi\theta_{i1}t + j\pi\theta_{i2}t^2} \end{aligned} \quad (3)$$

where $\Theta_i(t) = j2\pi\theta_{i1}t + j\pi\theta_{i2}t^2$. θ_{i2} denotes the signal modulated frequency, and θ_{i1} denotes frequency carrier, $[\theta_{i1}, \theta_{i2}]$ is the parameters vector which needs to be identified. a_i denotes the signal amplitude. T_i is the time scale which represents the time width of ith signal.

The problem can be expressed as finding a proper $(\hat{\alpha}_i, \hat{u}_i)$ to make the target function $|X_{\alpha}(u)|^2$ maximum where $X_{\alpha}(u)$ is the discrete fractional Fourier transformation of $x_i(t)$ at the order α . That is denoted by:

$$(\hat{\alpha}_i, \hat{u}_i) = \underset{(\alpha,u)}{\operatorname{argmax}}(|X_{\alpha}(u)|^2) \quad (4)$$

Substituting the parameter of Eq. (4) into Eq. (3), the FrFT of $x(t)$ is denoted by:

$$X_{\alpha}(u) = a_i T_i A_{\alpha} \mathrm{sinc}[\pi(u\csc\alpha - \theta_1)T_i] e^{j\pi u^2 \cot\alpha} \quad (5)$$

where $T_i = NT_s$. $|X_{\alpha}(u)|$ is denoted by:

$$| X_{\alpha}(u) | = \frac{a_i T_i \text{sinc}[\pi(u\csc\hat{\alpha}_i - \theta_1) T_i]}{\sqrt{\sin\hat{\alpha}_i}} \quad (6)$$

The relationship between $\hat{\theta}_{i1}, \hat{\theta}_{i2}, \hat{u}_i$ and $\hat{\alpha}_i$ is as the following:

$$\begin{cases} \hat{\theta}_{i2} = -\cot\hat{\alpha}_i \\ \hat{\theta}_{i1} = \hat{u}_i \csc\hat{\alpha}_i \\ \hat{a}_i = \dfrac{| X_{\hat{\alpha}_i}(\hat{u}_i) | \sqrt{\sin\hat{\alpha}_i}}{T_i} \end{cases} \quad (7)$$

3 Parameter Estimation of Chirp Signal Associated With FrFT

The order estimation can be interpreted as searching in a two dimensional plane at (u, α) . A continuous rotation angle α is used as a self-variable to get an energy distribution. The distinct energy peak will appear at the optimal order of the DFrFT of chirp signal. We can make a peak search according to the maximum value in the plane by DFrFT on the composite signal.

We demonstrate two approaches to estimate the parameters of the single-chirp and multi-chirp respectively. A high precision search based on DE algorithm is proposed to deal with the single chirp signal. By this method, we can obtain the order a of single chirp signal easily. To deal with the multi-component chirp signals, the method based on DE algorithm is proposed which contains two steps: rough search and high precision search. Firstly, the rough search gets the local optimal solutions in a low accuracy. Then based on the results from the rough search, we obtain the high optimal solutions by high precision search.

3.1 Parameters Estimation of Single Chirp

The high precision search algorithm based on DE to estimate the order of single chirp signal is expressed as follows:

1) Population Initialization. Get the target vector α_x which has *NP* elements. Each element in α_x is a random number from 0 to π.

2) Get the Maximum Value from Parent Individual. By the α_xth-order FrFT, the maximum value of chirp signal with each α can be obtained in the FrFD. The maximum value mentioned above represents the energy aggregation degree in the FrFD, and they have a significantly positive correlation.

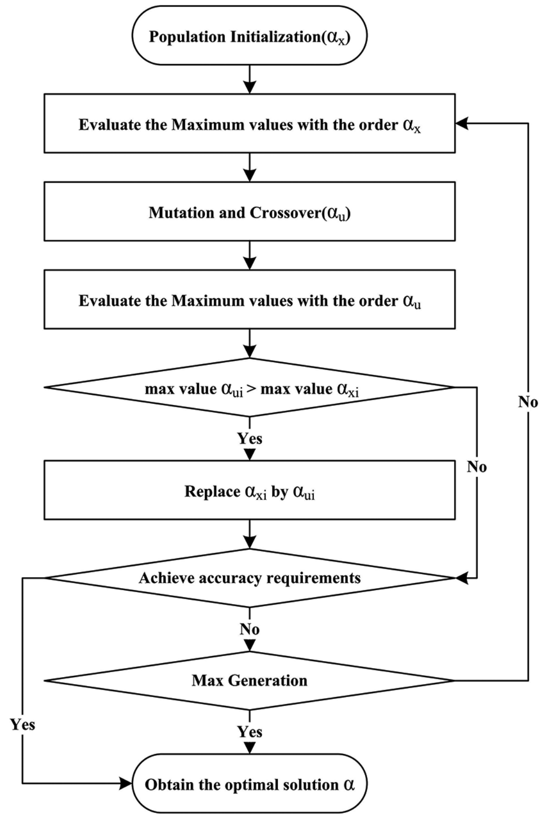
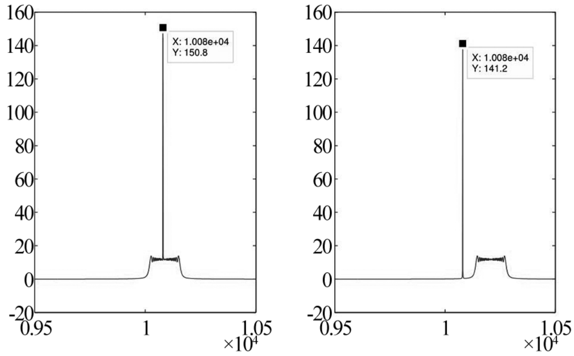


Fig. 2 Flowchart of DE algorithm for single component chirp

3) Mutation and Crossover. Create the mutant vector α_v by mutation. Then obtain the trail vector α_u after the crossover of target vector α_x and mutant vector α_v . Here we choose the mutation strategy: DE/best/1 because this strategy has fast convergence rate on solving single peak value problem[20, 21].

4) Selection Operator. Do the FrFT with the order α_u , and get their maximum values which is the same with step2. Then choose the better α from the parent individual and the new medium individual which has larger maximum value as α_x . By this step, the better results are selected to retain and the other results are removed.

5) Cycle Operator. Repeat step 2, 3, 4 until the difference between the maximum and minimum of α_x is smaller than a precision value given before or the max iteration G is reached. We choose the mean value of the elements in vector α_x as the optimal solution. Thus the best α of one component of the chirp signal can be obtained.



(a) Two signals with intersection (b) Two signals without intersection

Fig. 3 The intersected situation between two signals

Fig.3 gives the amplitude of the FrFT of two chirp signals. It is observed the peak value of one component is a superposition of the two components energy when the two signals have the intersection in the FrFD. In this situation, one component will affect another component. If we filter the strong component to find the order of the weak component, the energy of weak component will lose which may bring error to the result. Approaches such as the fine search algorithm and the Quasi-Newton method have the same problem, they only obtain the global optimal solution at a time. Although the methods can solve the problem of parameter estimation of single component chirp signals, they must use filter to get the other solutions when dealing with multi-component chirp signal. To solve the problem with multi-component chirp signal, we combine the rough search and the high precision search.

Comparing with these methods, differential evolution is more efficient on calculation. DE algorithm can converge to many local optimal solutions including the global optimal solution at a time without using filter. Thus the influence of window functions does not exist in our proposed method and the accuracy of parameter estimation is better than these traditional methods in some degree.

3.2 Parameters Estimation of Multi-Chirp signals

As we know the DE algorithm will finally converge to the global optimal solution. Using the feature that the local optimal solutions appear before the global optimal solution during the convergence process, we can get the local optimal solutions as the order of each component of the chirp signal in a low accuracy. We insert several steps into the process of high accuracy search algorithm and get a rough search algorithm. The rough search algorithm can be expressed as following:

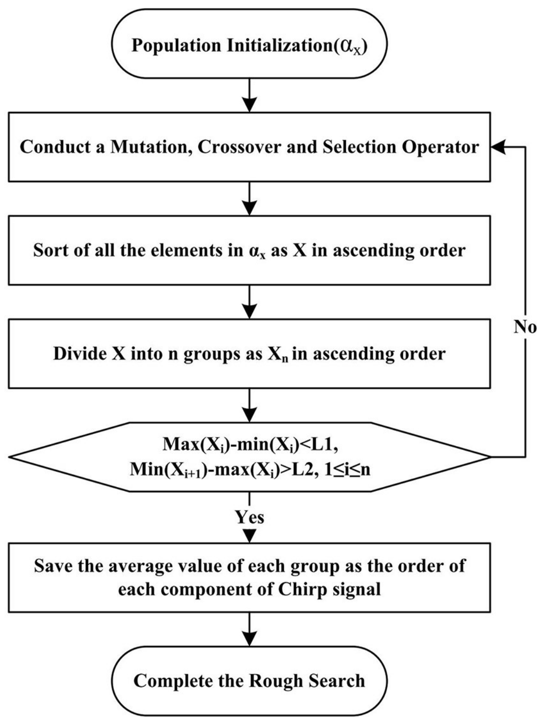


Fig. 4 Flowchart of Rough Search based on DE

1) Population Initialization. Get the target vector α_x which has *NP* columns. Each element in α_x is a random number from 0 to π.

2) Preparation Steps. Do step 2, 3, 4 in high accuracy search algorithm and get the α_x to prepare for the next steps. Here we choose the mutation strategy: DE/rand/1 because this strategy has good

global exploration ability on solving multi-peak value problem.

3) Sort and Divide. Sort all the elements of α_x in ascending order as vector X. Then divide the elements in X into N vectors X_i in ascending order. Thus we get a set of vectors X_i in strict ascending order.

4) Circulation and Judgement. Define the maximum element in vector X_i as $\max(X_i)$. Define the minimum element in vector X_i as $\min(X_i)$. Define L_1 and L_2 as two constants.

If the following condition:

$$\mathrm{Max}(X_i) - \mathrm{Min}(X_i) < L_1 \ , 1 \le i \le N \text{ and}$$
$$\mathrm{Min}(X_{i+1}) - \mathrm{Max}(X_i) > L_2 \ , 1 \le i \le N-1 \quad (8)$$

is satisfied, the mean value of the elements in each X_{i} is the result α we needed and we finally get N optimal solutions. Otherwise repeat step 2, 3 until the condition mentioned above is satisfied.

Based on the rough search, the values of orders of multi-component chirp signal are determined. The distribution of the *NP* individuals during the rough search process is shown in section 4. The high accuracy order can be found by the high precision search based on the DE algorithm in the neighborhood of rough searched orders.

The modulated rate of the sampled signal with time measured in seconds and frequency (Hz) is denoted by follows:

$$\hat{\alpha}_i = -\tan - 1\left(\frac{\delta f/\delta t}{\hat{\theta}_2}\right) \quad (9)$$

where δf is the frequency resolution $\delta f = 2\pi f_s/N$, f_s is the sampling rate, N is the number of sampling points. Here δt is the time resolution, $\delta t = 1/f_s$. The resolution of the α-order FrFD is as the following:

$$\Delta\alpha = \sqrt{T_s^2\cos^2\alpha + \frac{4\pi^2}{N^2T_s^2}\sin^2\alpha} \quad (10)$$

Hence,

$$\begin{cases} \hat{\theta}_{i2} = -\dfrac{2\pi f_s^2}{N}\cot\hat{\alpha}_i \\ \hat{\theta}_{i1} = \dfrac{2\pi f_s}{N}\hat{u}_i\csc\hat{\alpha}_i \\ \hat{a}_i = \dfrac{2\pi f_s \mid X_{\hat{\alpha}_i}(\hat{u}_i) \mid \sqrt{\sin\hat{\alpha}_i}}{N} \end{cases} \quad (11)$$

The accuracy of the α is fixed to $\Delta\alpha$, the quantization error is denoted by:

$$\begin{aligned} e &= \hat{\theta}_{i2} - \theta_{i2} \\ &= -\cot\hat{\alpha}_i + \frac{f_s^2}{N}\cot(\hat{\alpha}_i \pm \Delta\alpha) \end{aligned} \quad (12)$$

It is obvious that the accuracy of the estimation depends on the real modulated rate, the sampling rate and the number of sampling points. It has a positive relationship with chirp-rate, the bigger the modulated rate θ_{i2} is, the bigger the error is. When the length of the time is fixed. There is a tradeoff between length of the FrFT and the resolution. The smaller sampling rate and larger number of sampling points will result in the higher resolution. The sampling rate must be large enough to satisfy the Nyquist rate. The number of sampling points is defined as the length of DFrFT. Increasing the number of sampling points will decrease the error, which means that it will be easier identify chirp signals with the same center frequency than before. The number of sampling points must be big enough to ensure a reasonable linear approximation to the signal over the time duration.

4 Numerical Simulation and Performance Analysis

We separate the simulation into two parts, including the single chirp signal simulation and multi-component chirp signal simulation. The results are evaluated by the accuracy and computational load.

$$\mathrm{MSE} = 10\log\frac{\sum_{i=1}^{N}(\hat{\alpha}_i - \bar{\hat{\alpha}})^2}{(N-1)} \quad (13)$$

The MSE (Mean Squared Error) is used to evaluate the accuracy, The number of the FrFT is used to access complexity of the algorithm, both the MSE and the computational complexity are obtained by mean of the 100 times trials with mutative SNR.

4.1 Single Chirp Signal Simulation

We use the high precision search based on DE algorithm to deal with the single chirp signal. In this single chirp signal case, $\theta_1 = 500$, $\theta_2 = 10$. Fig.5 gives a distribution of the chirp signal with variable (u,α) in the FrFD, in which the sampling rate is $f_s = 100$Hz and the number of sampling points is 400. Mean and variance of the additive Gaussian noise is zero and 1. The SNR of input chirp signal is 1dB. Fig.6 shows the relationship between the SNR and MSE. We make 100 trials to find that the statistics average number of generation is 11. Estimated value is $\alpha = 1.6109$ which is a little different due to the effects of both discrete sampling and noise background.

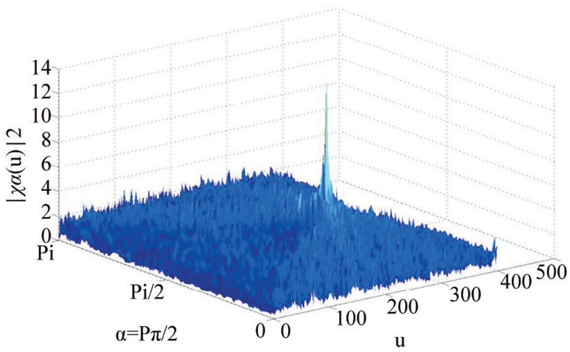


Fig. 5 The energy distribution of the single chirp signal

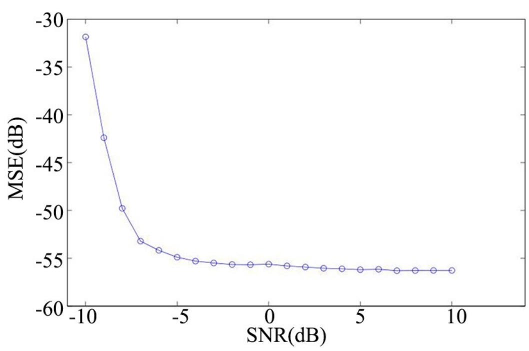


Fig. 6 MSE of order of single chirp signal by high precision search

4.2 Multi-component Chirp Signal Simulation

Fig. 7 gives an distribution of the chirp signal with variable (u,α) in the FrFT domain, in which the sampling rate is $f_s > B_i t$ and the number of sampling points is 400. Mean and variance of the additive Gaussian noise is zero and 1 respectively. The SNR of input chirp signal is 1dB. The energy of strong component of input chirp signal is two times than that of weak component. It is observed that the strong signal may cover up the weak signal. The rough search method which based on DE algorithm can obtain the order of the two components of the signal roughly without eliminating the strong signal.

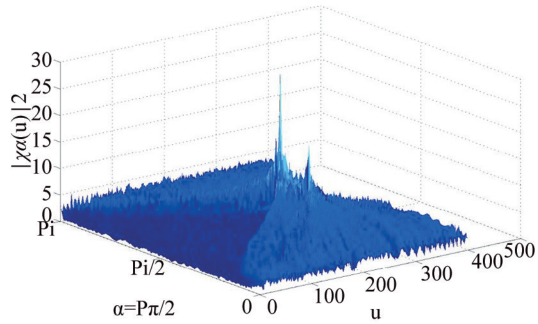


Fig. 7 The energy distribution of the multi-chirp signal

Fig.8 shows the relationship between the SNR and MSE, the order1 represent the strong component and the order2 represent the weak component. Fig.9 shows the MSE of orders while the energy of strong signal is M times more than that of weak signal ($M = a_1/a_2$). The distance between α_1 and α_2 also affects the accuracy and MSE of the orders obtained by rough search. We make a 100 trials to find that the statistics average number of generation of rough search is 7. And the results of the simulation show the mean number of the order α is 1.6109 and 1.9557.

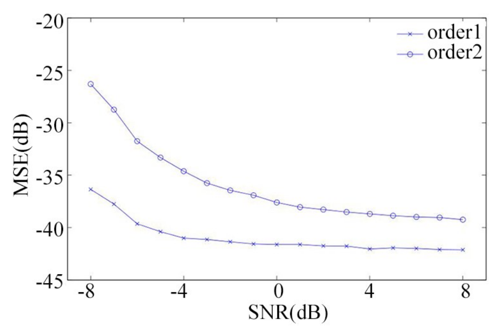


Fig. 8 MSE of α of two-component chirp signal by rough search

As Fig.10 shows, the distribution of the 20 individuals obtained by rough search is determined by the search generation. With the search generation increasing, the results gather to the two local optimal solutions and the accuracy of the results becomes higher. The degree of aggregation also relates to the chirp rate, the energy and the SNR of the chirp signal.

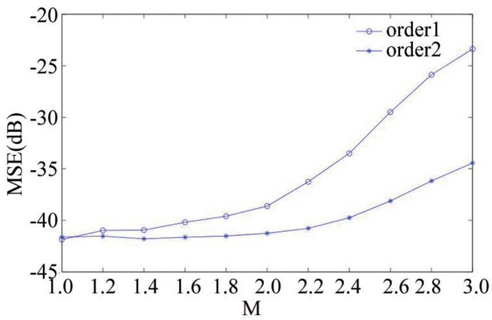


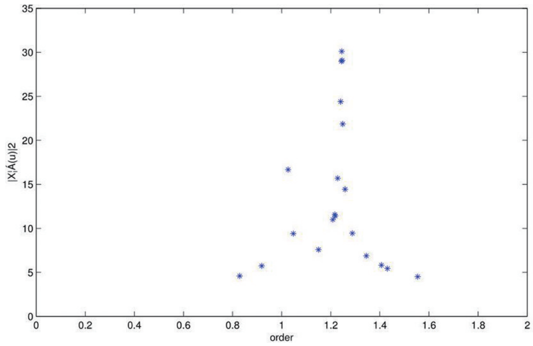
Fig. 9 MSE of α of two-component chirp signal under different multiples ($M = \alpha_1/\alpha_2$)

In most practical applications, the frequency of the chirp signals changes within a certain period of time, but the start point is not fixed without changing. Sometimes it may not get the full information of the signal because the finite time sampling and FrFT only use part of the signal, the other part of signal need to be sampled at next period.

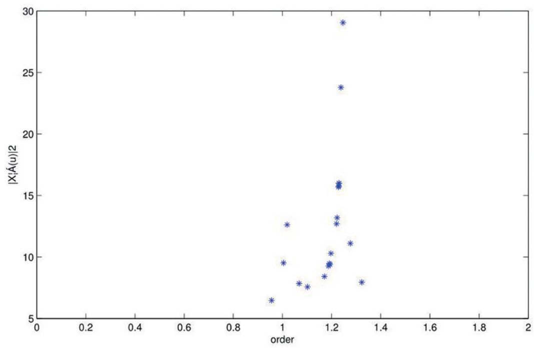
4.3 Performance Analysis

Here we use the multi-component chirp signal mentioned above to do the simulation with its SNR= 1dB. According to the simulation results, the rough search by our method costs 1.55 seconds on average by our computer. And combined with the high accuracy search, the whole algorithm cost 2.75 seconds on average and then we have high accuracy solutions. While using the algorithm proposed by Zang[14], we find only the first step of searching for the best orders which corresponds to the rough search in our paper costs 3.37 seconds on average. Thus in the case of the same accuracy, the algorithm based on DE we proposed can reduce time consuming apparently.

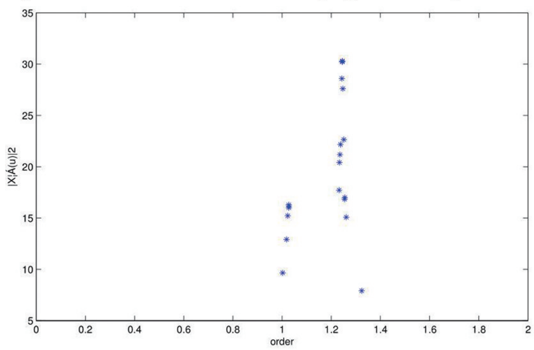
From the figures above we can see, when we use high precision search to deal with the single chirp signal, the result shows that it can suppress the influence of white Gaussian noise. If the SNR is more



a) The distribution of the 20 individuals by 3 generation rough search



b) The distribution of the 20 individuals by 5 generation rough search



c) The distribution of the 20 individuals by 7 generation rough search

Fig. 10 The distribution of order under different number of steps.

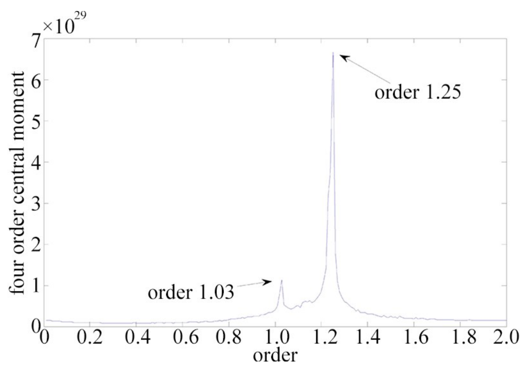


Fig. 11 The two solutions obtained by the method from Zang[14]

than -10dB, the algorithm have high efficiency and accuracy. Also the searching generation is related to the accuracy we need and the number of sampling points.

In the rough searching part, the white Gaussian noise has influence on the accuracy of the orders we get, especially the order of weak component of the signal. Also the difference between the strong signal and weak signal in terms of energy has influence on the MSE of results. This phenomenon indicates that while the SNR of signal is low or the difference of energy between strong and weak signals is great, the performance of this method will become bad. Although we could improve the accuracy of the determination requirements and increase the generation of DE search to make the performance better, the negative effect still exist.

The number of multiplications for the DFrFT with the method[23] is $(N^2+1)/2$ and the number of additions is $(N^2+1)/2+2N-2$ as the sampling number N is odd.

5 Conclusion

In this paper, a novel parameter estimation method for multi-component chirp signals is presented. The proposed method is based on the discrete fractional Fourier transformation (DFrFT) and the differential evolution (DE) algorithm. The results of the simulation have shown the proposed method can provide significant interference suppression. Through the whole process we can see the algorithm we proposed is different from the method using filters. Without using non-ideal filter, the weak component of the signal will have no energy loss. Thus the algorithm guarantees the precious of the result. Compare with the improved strategies based on WVT, our method completely avoids the cross terms. Thus the loss of precision caused by cross terms is not exist when we process low SNR multi-component chirp signals. The proposed approach without accuracy degradation which is instead of a conventional fine search algorithm can not only improve the estimation accuracy, but also reduce time consuming, and it is relative easy to achieve. In addition, sampling duration and sampling rate also play important roles in parameter estimation.

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