

Bistable Piezoelectric Flutter Energy Harvesting with Uncertainty

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Abstract: The analytical formulation of piezoelectric flutter energy harvesting using a bistable material, while considering uncertainties in the model is presented in this paper. Bistable laminates provide the advantage of large deflection due to the nonlinear snap-through characteristics when exposed to external loading, and can therefore provide a suitable base for piezoelectric material in energy harvesting applications. A piezoelectric material that is bonded on the surface of bistable laminates, subjected to external loading, generates large strains and hence relatively higher electrical output energy, in comparison with the case where piezoelectric material is bonded on a regular surface, with analogous loading conditions. Although information regarding the external loading, material characteristics of the bistable laminate and the piezoelectric material, boundary conditions, and overall electrical circuit efficiency can be defined for analytical purposes, the exact model of the system is not readily accessible. The unavoidable uncertainties in the material, loading, and efficiency of a complex system call for a probabilistic approach. Hence, this paper provides a formulation that considers uncertainty bounds in obtaining a realistic model. Optimal Uncertainty Quantification (OUQ) is used in this paper, which takes into account uncertainty measures with optimal bounds and incomplete information about the system, as a well-defined optimization problem according to maximum probabilities, subjected to the imposed constraints. The OUQ allows the inspection of the solution for a span of uncertain input parameters, as a reliable and realistic model.

Key words: Bistable laminates, piezoelectric energy harvesting, flutter energy harvesting, optimal uncertainty quantification, self-powered systems.

1 Introduction

Energy independent systems are realized by implementing energy harvesting techniques, in the development of self-powered sensors and actuators^{[1]-[4]}, using electromagnetic^{[5]-[9]}, piezoelectric^{[10]-[12]}, or electrostatic^[10] energy conversion mechanisms. This is a power supply solution particularly for low-powered electronics, (e.g. with application in wireless sensor networks and monitoring devices), and renewable energy sources^{[1]-[4], [13]}.

Various techniques have been employed by researchers to make use of such piezoelectric energy harvesting systems with optimum output results. There is an extensive literature on energy harvesting, and some examples are indicated in the following brief review.

Techniques that allow improved output efficiency

are included in multiple-frequency energy harvesting across a wide band^{[14]-[15]}. For example, external magnets are used to provide stiffness nonlinearities^[16], which tune the operating frequencies, and broaden the frequency bandwidth. Also, temperature-dependent bistable laminates are used^[17], with significant power outputs over a wide bandwidth by utilizing square bistable laminates^{[18]-[21]}.

Static stable models are obtained by identifying the coefficients of assumed polynomials and minimizing the total strain energy. This has been validated experimentally and also using finite element analysis for square and rectangular laminates^{[21]-[23]}. The double-well potential energy dynamic model of the system^[24] predicts the outputs for any given vibration pattern. Such modeling has been applied to bistable laminates of trapezoidal and triangular plan-

forms ^[25] for morphing applications, and have been modelled analytically as tapered cantilever configurations ^[26], and in arbitrary shapes ^{[27]-[28]}.

The unavoidable uncertainties in the material, loading, and efficiency of a complex bistable piezoelectric flutter energy harvester call for a probabilistic approach. Hence, this paper offers a formulation that considers uncertainty bounds in obtaining a realistic model. Optimal Uncertainty Quantification (OUQ) ^[29] is used in this paper, which takes into account uncertainty measures with optimal bounds and incomplete information about the system, as a well-defined optimization problem according to probability maximization, subjected to the imposed constraints. In this paper, an analytical model of nonlinear piezoelectric bistable laminate is introduced. Next, the dynamic model of the system subjected to aerodynamic loading is presented. Finally, the stochastic model of the system with bounded constraints is formulated, which can inspect the solution for a span of uncertain input parameters, as a reliable and realistic model.

2 Nonlinear Dynamics of Piezoelectric Bistable Laminates

An analytical, geometrically nonlinear model for piezoelectric bistable laminates with nonlinear electromechanical coupling ^{[27]-[28]} is presented in this section.

2.1 Constitutive equations for piezoelectric plates

The assumptions made in modeling the nonlinear piezoelectric bistable laminates are as follows ^{[27]-[28]}:

- 1 Bistable laminates are considered as plate structures.
- 2 Only the laminate is cured at high temperature and then the piezoelectric layer is attached.
- 3 The piezoelectric material is polarized along its thickness.
- 4 The Kirchhoff plate theory is applicable.

The constitutive equations for piezoelectric material may be expressed as

$$\begin{cases} \sigma = C\varepsilon - eE \\ D = e\varepsilon + \eta E \end{cases} \quad (1)$$

where σ and ε are stress and strain fields, respectively, C is the elasticity matrix, e is the electro-mechanical coupling coefficients matrix, E is the electrical field vector, D is the electrical displacement vector, and η is the permittivity components matrix. Based on the Kirchhoff theory, with $\sigma_{33} = 0$, for plate structures, the constitutive relation can be given by,

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \\ D_3 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{16} & e_{13} \\ C_{21} & C_{22} & C_{26} & e_{23} \\ C_{61} & C_{62} & C_{66} & 0 \\ e_{31} & e_{32} & 0 & -\eta_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \\ -E_3 \end{bmatrix} \quad (2)$$

The shear forces perpendicular to the mid plane are assumed to be negligible. Axial forces (N) and bending moments (M) are given by the expressions

$$\begin{cases} N = (N_{11} & N_{22} & N_{12})^T = \int_{t_1^p}^{t_2^p} \sigma^0 dz \\ M = (M_{11} & M_{22} & M_{12})^T = \int_{t_1^p}^{t_2^p} z \sigma dz \end{cases} \quad (3)$$

where $t_2^p - t_1^p$ is the thickness of the piezoelectric layer and σ^0 and σ are the membrane and bending parts, respectively, of the section stress, which is given by,

$$\sigma^0 = C \varepsilon^0 ; \sigma = z C \kappa \quad (4)$$

where ε^0 and κ are in-plane strains and curvatures.

By substituting (2) into (3), the constitutive relations for a cross-ply laminate in a general and compact form may be expressed as,

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix} - (t_2^p - t_1^p) \begin{bmatrix} I_3 \\ \frac{(t_2^p + t_1^p)}{2} I_3 \end{bmatrix} \begin{bmatrix} e_{31} \\ e_{32} \\ 0 \end{bmatrix} E_3 \quad (5)$$

where A , D and B denote the membrane (extensional), bending and coupling stiffness matrices, respectively, and I_3 is the identity matrix. By integrating both sides of the sensing equation over the thickness

$$(t_2^p - t_1^p) D_3 = [e_{31} \quad e_{32} \quad 0] (t_2^p - t_1^p) \begin{bmatrix} I_3 & \frac{(t_2^p + t_1^p)}{2} I_3 \end{bmatrix} \begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix} + \varepsilon_{33} E_3 (t_2^p - t_1^p) \quad (6)$$

By cancelling the piezoelectric thickness from bothsides of Equation (6),

$$\begin{cases} \bar{N} = Q \bar{\varepsilon} - t^p e' E_3 \\ D_3 = e'^T \bar{\varepsilon} + \varepsilon_{33} E_3 \end{cases} \quad (7)$$

where $\bar{N} = \begin{bmatrix} N \\ M \end{bmatrix}$, $\bar{\varepsilon} = \begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix}$, $Q = \begin{bmatrix} A & B \\ B & D \end{bmatrix}$, $e' = \begin{bmatrix} I_3 \\ \frac{t}{2} I_3 \end{bmatrix} \begin{bmatrix} e_{31} \\ e_{32} \\ 0 \end{bmatrix}$, and $\bar{t} = t_2^p + t_1^p$, $t^p = t_2^p - t_1^p$.

Seperation of the linear and nonlinear parts of the strain field leads to

$$\bar{\varepsilon} = \bar{\varepsilon}^L + \bar{\varepsilon}^{NL}; \quad \bar{\varepsilon}^L = \begin{bmatrix} \varepsilon^{0L} \\ \kappa \end{bmatrix}; \quad \bar{\varepsilon}^{NL} = \begin{bmatrix} \varepsilon^{0NL} \\ 0 \end{bmatrix} \quad (8)$$

Rewriting (7) in terms of linear and nonlinear parts,

$$\begin{cases} \bar{N} = Q \bar{\varepsilon}^L + Q \bar{\varepsilon}^{NL} - t^p e' E_3 \\ D_3 = e'^T \bar{\varepsilon}^L + e'^T \bar{\varepsilon}^{NL} + \varepsilon_{33} E_3 \end{cases} \quad (9)$$

2.2 Constitutive equations for bistable laminate

The constitutive equation for bistable laminates may be expressed as follows

$$\bar{N}^s = Q^s \bar{\varepsilon}^s \quad (10)$$

where the superscript s stands for substructure. This equation in terms of linear and nonlinear parts of the relation, can be written as

$$\bar{N}^s = Q^s \bar{\varepsilon}^{L,s} + Q^s \bar{\varepsilon}^{NL,s} \quad (11)$$

The in-plane strain field and out of plane displacement component (deflection) are approximated based on Hyer's model ^[1] as,

$$\begin{bmatrix} \varepsilon_x^0 & \varepsilon_y^0 & \gamma_{xy}^0 \end{bmatrix} \approx \bar{\varphi} d + f \approx \varepsilon^{0L} + \varepsilon^{0NL} \quad (12)$$

where $\bar{\varphi}$ is the approximation function matrix for linear strains and f is the nonlinear part of the approximation.

Using in-plane strain field approximation in (12), $\bar{\varepsilon}^L$ and $\bar{\varepsilon}^{NL}$, from Equation (8) may be ex-

pressed in terms of the elongation coefficients d and the curvature coefficients a in a matrix form as follows:

$$\bar{\varepsilon}^L = \begin{bmatrix} \varepsilon^{0L} \\ \kappa \end{bmatrix} \approx \begin{bmatrix} \bar{\varphi} & 0 \\ 0 & -I_3 \end{bmatrix} \begin{bmatrix} d \\ a \end{bmatrix} = \Phi X \quad (13)$$

$$\bar{\varepsilon}^{NL} = \begin{bmatrix} \varepsilon^{0NL} \\ 0 \end{bmatrix} \approx [00f000]^T = \begin{bmatrix} f \\ 0 \end{bmatrix} = \Psi \quad (14)$$

Now, $\bar{\varepsilon}$ in the constitutive relations is substituted by its approximated values as,

$$\bar{\varepsilon} = \begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix} = \bar{\varepsilon}^L + \bar{\varepsilon}^{NL} = \Phi X + \Psi \quad (15)$$

2.3 Derivation of the governing dynamic equations

To satisfy the dynamic equilibrium of the bistable laminate with bonded piezoelectric elements, according to Hamilton's principle, the potential energy of the structure under dynamic conditions is minimized as follows:

$$\int_{t_1}^{t_2} (\delta T - \delta U + \delta W_{ie} + \delta W_{nc}) dt = 0 \quad (16)$$

The energy terms in Equation (16) for the nonlinear piezoelectric composite plate can be identified. The total strain energy of the system may be written as,

$$U = U^s + U^p = U^{m,s} + U^{th,s} + U^{m,p} + U^{e,p} \quad (17)$$

where, superscripts s , p , m , th , and e stand for substructure, piezoelectric, mechanical, thermal and electrical strain energies, respectively.

3 Piezoelectric Flutter 2DOF

A simplified two-degree-of-freedom (2DOF) lumped parameter model of piezoelectric flutter energy harvesting is discussed now. It will be further expanded to a continuous model associated with the bistable laminate as a basis of the piezoelectric part, in the next section. The degrees of freedom for the aeroelastic vibration of a piezoelectric energy harvester ^{[31]-[33]}, when exposed to uniform incompressible potential flow, can be defined by the plunge displacement or translation h , and the pitch displacement or rotation α as shown in Fig. 1, which are de-

finied using aircraft wing flutter analysis. Also, h and α are the displacements of a base plate that is responsible for the input excitation of the bounded piezoelectric material. The optimization problem is expressed as

$$\int_{t_2}^{t_1} (\delta T - \delta U + \delta W_{ie} + \delta W_{nce} + \delta W_{nca} + \delta W_{sd}) dt = 0 \quad (18)$$

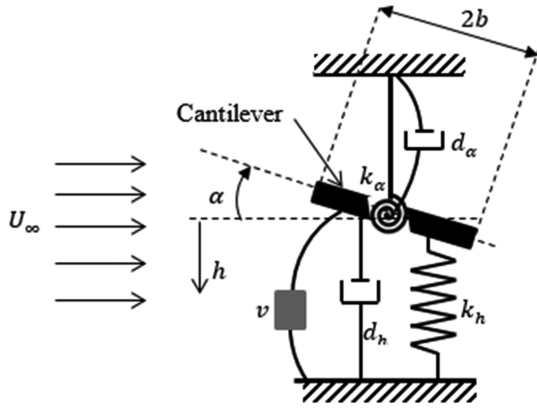


Fig. 1 A 2DOF model for aeroelastic vibration [2].

where T and U are the total kinetic and potential energies per unit length, W_{ie} denotes the internal electrical energy per unit length, W_{nce} is the virtual work component of the non-conservative electrical charge, W_{nca} is the virtual work component of the aerodynamic loads, and W_{sd} is the virtual work of the structural damping.

The kinetic energy of the system per length can be written as

$$T = \frac{1}{2} m (\dot{h}^2 + 2 x_\alpha b \dot{h} \dot{\alpha}) + \frac{1}{2} m_f \dot{h}^2 + \frac{1}{2} I_p \dot{\alpha}^2 \quad (19)$$

where m is the airfoil mass per unit length, m_f is the fixture mass per unit length connecting the airfoil to the plunge springs, x_α is the chord-wise offset from the reference point to the centroid, b is the semi-chord length, and I_p is the moment of inertia per unit length about the reference point. The potential energy of the system can be given by

$$U = \frac{1}{2} k_h h^2 + \frac{1}{2} k_\alpha \alpha^2 - \frac{1}{2} \frac{\theta}{l} h v \quad (20)$$

where k_h is the stiffness per unit length associated with the plunge degree of freedom, k_α is the stiffness in pitch degree of freedom, θ is the electromechanical coupling term, l is the span length of the cantilever, and v is the voltage. The internal electrical energy is

$$W_{ie} = \frac{1}{2} \frac{C_p^{eq}}{l} v^2 + \frac{1}{2} \frac{\theta}{l} h v \quad (21)$$

where C_p^{eq} is the equivalent capacitance of the piezoelectric material.

The virtual work components can be determined as

$$\delta W_{nce} = \frac{Q}{l} \delta v \quad (22)$$

$$\delta W_{nca} = -L \delta h + M \delta \alpha \quad (23)$$

$$\delta W_{sd} = -d_h h \delta h - d_\alpha \alpha \delta \alpha \quad (24)$$

where Q is the output of the electrical charge, L is the aerodynamic lift per unit length, M is the aerodynamic pitching moment per unit length, d_h is the structural damping coefficient per unit length for the plunge DOF and d_α is the structural damping coefficient per unit length for the pitch DOF.

By assuming a harmonic response, it defines $h = \bar{h} e^{j\omega t}$, $\alpha = \bar{\alpha} e^{j\omega t}$, $v = \bar{v} e^{j\omega t}$, $L = \bar{L} e^{j\omega t}$, $M = \bar{M} e^{j\omega t}$. Using the Lagrange equations with the harmonic response assumption, the Equations (19)-(24) will lead to the complex eigenvalue problem below [31]

$$\begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} \bar{h} \\ \bar{\alpha} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (25)$$

where

$$a_{11} = \left[\beta + \frac{l_h}{\mu} \kappa(\omega) - \sigma^2 (1 + j \gamma_h) \lambda \right],$$

$$a_{21} = \left(x_\alpha + \frac{l_a}{\mu} \right),$$

$$a_{12} = \left(x_a + \frac{m_h}{\mu} \right),$$

$$a_{22} = \left[r^2 + \frac{m_\alpha}{\mu} r^2 (1 + j \gamma_\alpha) \lambda \right],$$

and, λ , σ , r , μ , and β are dimensionless

terms denoting complex eigenvalue ($\lambda = (\omega_\alpha/\omega)^2$), frequency ratio ($\sigma = \omega_h/\omega_\alpha$), radius of gyration ($r = \sqrt{I_p/m b^2}$), ratio of the airfoil to the affected air mass ($\mu = m/\pi \rho_\infty b^2$), and mass ratio ($\beta = (m + m_f)/m$), respectively, where $\omega_h = \sqrt{k_h/m}$, $\omega_\alpha = \sqrt{k_\alpha/I_p}$, ρ_∞ is the free-stream air mass density, $\gamma_h = d_h \omega/k_h$, $\gamma_\alpha = d_\alpha \omega/k_\alpha$, and l_h , l_α , m_h , m_α are the components of the aerodynamic loads, given by

$$l_h = 1-j \frac{2}{k} C(k) \quad (26)$$

$$l_\alpha = -a-j \frac{1}{k} \frac{2}{k^2} C(k) - j \frac{2}{k} \left(\frac{1}{2} - a \right) C(k) \quad (27)$$

$$m_h = -a + j \frac{2}{k} \left(\frac{1}{2} + a \right) C(k) \quad (28)$$

$$m_\alpha = \frac{1}{8} + a^2 - j \frac{1}{k} \left(\frac{1}{2} - a \right) + \frac{2}{k^2} \left(\frac{1}{2} + a \right) C(k) + j \frac{2}{k} \left(\frac{1}{4} - a^2 \right) C(k) \quad (29)$$

where α is the dimensionless location of the reference point, k is the reduced frequency, $C(k)$ is the The odorsen function, and $H_n^{(2)}(k)$ are Hankel functions, which are given by

$$k = \frac{b\omega}{U} \quad (30)$$

$$C(k) = \frac{H_1^{(2)}(k)}{H_1^{(2)}(k) + j H_0^{(2)}(k)} \quad (31)$$

$$H_n^{(2)}(k) = J_n(k) - j Y_n(k) \quad (32)$$

$J_n(k)$ is the Bessel function of the first kind, and $Y_n(k)$ are Bessel function of the second kind.

The term $\kappa(\omega)$ is

$$\kappa(\omega) = \frac{j \theta^2}{\omega m l \left(j \omega C_p^{eq} + \frac{1}{R_l} \right)} \quad (33)$$

where R_l is the external electrical load resistance and $Q = v/R_l$.

Iterative eigenvalue procedure is employed (e.g. using the P-K Method) to obtain the eigensolution of (25) for $\tilde{h}$ and $\tilde{\alpha}$. Consequently $\tilde{v}$ is determined by

$$\tilde{v} = \frac{-j\omega\theta\tilde{h}}{j\omega C_p^{eq} + \frac{1}{R_l}} \quad (34)$$

The amount of electrical power can be calculated by $\tilde{v}^2/R_l$.

The two-degree-of-freedom lumped parameter model in this section is extended to the bistable piezoelectric continuous model in the next section.

4 Bistable Piezoelectric Flutter

The flutter analysis, presented in Section 3, is now incorporated in the continuous model of the bistable piezoelectric laminate, that was discussed in Section 2.

A piezoelectric energy conversion mechanism is shown in Fig. 2, where the excitation of the bonded piezoelectric material, on the bistable laminate, is due to aeroelastic flutter.

The bistable piezoelectric system is shown in Fig. 3, which can provide large deflections when subjected to external loading due to the bistable snap-through characteristics of the bistable laminate, as in Fig. 4.

The substructure (bistable laminate), and the piezoelectric strain energies in Equation (17), can be given by

$$U^s = U^{m,s} + U^{th,s} = \frac{1}{2} \int_{A^s} (\bar{N}^T - \bar{N}^{thT}) \bar{\epsilon} d A^s \quad (35)$$

$$U^p = U^{m,p} + U^{e,p} = \frac{1}{2} \int_{A^s} \bar{N}^T \bar{\epsilon} d A^p \quad (36)$$

By substituting $\bar{N}$ from Equation (9) it obtains,

$$U^s = U^{m,s} + U^{th,s} = \frac{1}{2} \int_{A^s} (Q \bar{\epsilon}^L + Q \bar{\epsilon}^{NL} - Q \alpha \Delta T)^T (\bar{\epsilon}^L + \bar{\epsilon}^{NL}) d A^s \quad (37)$$

$$U^p = U^{m,p} + U^{e,p} = \frac{1}{2} \int_{A^s} (Q \bar{\epsilon}^L + Q \bar{\epsilon}^{NL} - t^p e^T E_3)^T (\bar{\epsilon}^L + \bar{\epsilon}^{NL}) d A^p \quad (38)$$

The variation of the strain energy of the system can be expressed by

$$\delta U = \frac{\partial U}{\partial X} \delta X + \frac{\partial U}{\partial v} \delta v \quad (39)$$

where X is the in-plane deformations and curva-

tures vector and v is the voltage.

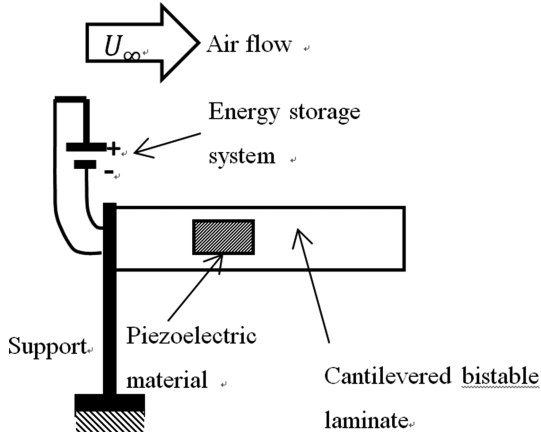


Fig. 2 Piezoelectric flutter energy harvesting.

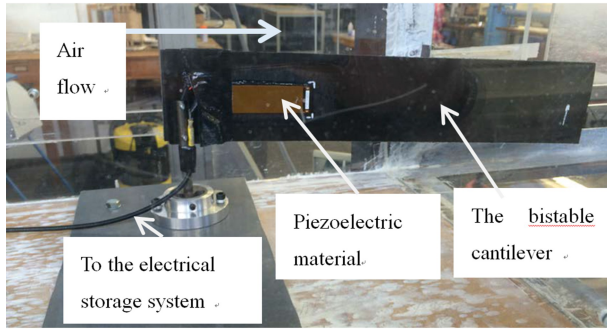


Fig. 3 The bistable piezoelectric system.

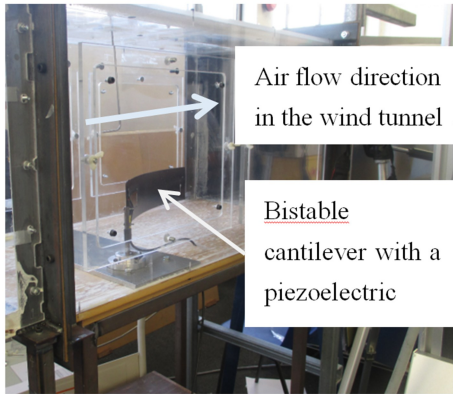


Fig. 4 Demonstration of bistable large deflection.

By combining all the energy terms, the governing dynamic equations can be found, where the energy terms are: the mechanical piezoelectric strain energy, electrical strain energy, mechanical sub-structure strain energy, thermal strain energy, piezoelectric internal electrical energy, kinetic energy of

the system, and non-conservative work. The governing dynamic equations, can be given by the Hamilton's principle in Equation (16) as multipliers of the virtual in-plane deformation and the curvature coefficients (δX) and the virtual voltage (δv). By factorizing δX and δv , their multipliers should be zero in the time interval t_1 to t_2 , and the governing equations are derived as follows:

$$\begin{bmatrix} M & 0 \\ 0 & J \end{bmatrix} \begin{bmatrix} \ddot{d} \\ \ddot{a} \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} d \\ a \end{bmatrix} + \begin{bmatrix} \theta_d \\ -\theta_a \end{bmatrix} v = \begin{bmatrix} L_{aero} \\ M_{aero} \end{bmatrix} \quad (40)$$

$$-\theta_d \dot{d} + \theta_a \dot{a} + \frac{C_p v}{t^p} + \frac{v}{R} = 0 \quad (41)$$

where d and a are the in-plane strain coefficient vector and the curvatures, respectively, M is the inertial mass matrix, J denotes the moment of inertia, and L_{aero} , and M_{aero} are the aerodynamic loads. The aerodynamic loads for a 2DOF problem are given in Equations (26) to (29).

$$K_{11} = \int_{A^s} \bar{\varphi}^T A^s \bar{\varphi} d A^s + \int_{A^p} \bar{\varphi}^T A^p \bar{\varphi} d A^p \quad (42)$$

$$K_{12}(a) = \int_{A^s} -a^T B^s \bar{\varphi} + f^T A^s \bar{\varphi} - \frac{1}{2} \Delta T \bar{\alpha} (A^s + B^s) d A^s + \int_{A^p} -a^T B^p \bar{\varphi} + f^T A^p \bar{\varphi} d A^p \quad (43)$$

$$K_{21}(a) = \int_{A^s} -\bar{\varphi}^T B^s a + \bar{\varphi}^T A^s f_{,a} d A^s + \int_{A^p} -\bar{\varphi}^T B^p a + \bar{\varphi}^T A^p f_{,a} d A^p \quad (44)$$

$$K_{22}(a) = \int_{A^s} a^T D^s - f^T B^s - a^T B^s f_{,a} + f^T A^s f_{,a} d A^s + \int_{A^p} a^T D^p - f^T B^p - a^T B^p f_{,a} + f^T A^p f_{,a} d A^p - \frac{1}{2} \int_{A^s} \Delta T \bar{\alpha} ((A^s + B^s) f_{,a} - (B^s + D^s)) d A^s \quad (45)$$

where $f_{,a}$ denotes $\frac{\partial f}{\partial a}$.

5 Uncertainty quantification for the piezoelectric bistable flutter model

The unavoidable uncertainties in the material,

loading, and efficiency of a complex system call for a probabilistic approach. Hence, this paper offers a formulation that considers uncertainty bounds in obtaining a realistic model. Optimal Uncertainty Quantification (OUQ) is used in this paper, which takes into account uncertainty measures with optimal bounds and incomplete information about the system, as a well-defined optimization problem according to probability maximization, subjected to the imposed constraints. The OUQ allows the inspection of the solution for a span of uncertain input parameters, as a reliable and realistic model. The analytical energy balance of the bistable piezoelectric system is given by the Equations (35) through (39). The strain energy of the system for the system, while considering uncertainties and the corresponding optimal bounds, can be given by OUQ. The parameters with uncertainties can include external loading, material characteristics of the bistable laminate and the piezoelectric material, boundary conditions, and the overall electrical circuit efficiency. Therefore, the corresponding parameters that are potentially uncertain in Equations (40) to (45) can be formulated by bounded probabilities.

If the output power of the piezoelectric system is given by P_o , then it can define $P_o: X \rightarrow \mathbb{R}$, $X \rightarrow P_o(X)$, with the probability of $\mathbb{P} \in M(X)$, where X and X denote the deterministic and the stochastic forms of the inputs, respectively. In a stochastic representation of a system, it is required that the probability of the power consumption function, $P_o(X)$ to be obtained for a bounded region of the uncertain parameters. If the system is required to supply a certain amount of desired power output, P_d , for instance, in achieving a self-powered scheme as an objective of the problem, failing of the system (which corresponds to $P_o < P_d$) can be formulated as

$$\mathbb{P}[P_d(X) \geq P_o] \leq \epsilon \quad (46)$$

which is referred to the scenario that: ‘the probability that the desired output power, P_d , is larger than the produced output power by the piezoelectric system, must be less than a defined ϵ , in order

for the system not to fail as a self-powered application. Suppose that the probability function is a member of admissible extremal scenarios A (or $(P_o, \mathbb{P}) \in A$), where A is defined as

$$A: \left\{ \begin{array}{l} p_o: X_1 \times \cdots \times X_n \rightarrow \mathbb{R} \\ (p_o, \mu) \mid \mu = \mu_1 \otimes \mu_2 \otimes \cdots \otimes \mu_n \\ \mathbb{E}_\mu[p_o] \leq P_{max} \end{array} \right\} \quad (47)$$

and n denotes the number of uncertain parameters to be considered in the analysis of the power/energy, for parameters defined as inputs to the power/energy function, and these inputs are denoted by X_i , $i = 1, \dots, n$. Also, $\mathbb{E}_\mu[p_o]$ is the bounded mean output power, μ_i is the probability measure of the input parameter X_i ($\mu_i \in P(X_i)$), and p_o is a possible output function of P_o associated with the corresponding inputs/parameters X_i . The original problem entails optimization over a collection of (p_o, μ) that could be $(P_o, \mathbb{P})$. P_{max} is the output upper bound of the piezoelectric physical limits in producing power/energy. X_i and μ_i parameters can be constrained values with corresponding lower and upper bounds for each i . The optimal bounds on the probability of power consumption according to the required performance can be described by the upper bound, $U(A)$ as

$$U(A) := \sup_{(p_o, \mu) \in A} \mu[p_d(X) \geq P_o] ,$$

The lower bound, $L(A)$, corresponding to the minimum required power consumption can be stated as

$$L(A) := \inf_{(p_o, \mu) \in A} \mu[p_d(X) \geq P_o]$$

Therefore, the optimal bounds of the problem can be given by

$$L(A) \leq \mathbb{P}[P_d(X) \geq P_o] \leq U(A)$$

The power output can be determined by solving the constrained optimization problem for extremal scenarios A , with the inputs/parameters are also constrained.

If any of the parameters that are used in obtaining the piezoelectric energy/power output, are considered for uncertainty analysis, the upper bound and lower bound for such parameters are introduced, and

the problem is solved accordingly, as an optimization problem with the bounds introduced as the problem constraints. For instance, parameters, v , M , J , L_{aero} , and/or M_{aero} , which are responsible in obtaining the output power/energy, can be introduced with well-defined upper and lower bounds in order to span the probability of such parameters being regarded as uncertain or incomplete information about the system/parameter. Then, in a mathematical form we can write, for instance,

$$\begin{aligned} M &\in X_1 := [M_{min}, M_{max}] \\ J &\in X_2 := [J_{min}, J_{max}] \\ L_{aero} &\in X_3 := [L_{aero min}, L_{aero max}] \\ M_{aero} &\in X_4 := [M_{aero min}, M_{aero max}] \\ v &\in X_5 := [v_{min}, v_{max}] \end{aligned}$$

In such case, the admissible set A may be given by

$$A: \left\{ \begin{array}{l} p_o: X_1 \times X_2 \times \dots \times X_5 \rightarrow \mathbb{R} \\ (p_o, \mu) \mid \mu = \mu_1 \otimes \mu_2 \otimes \mu_3 \otimes \mu_4 \otimes \mu_5 \\ \mathbb{E}_\mu [p_o] \leq P_{max} \end{array} \right\} \quad (48)$$

As seen in this example, the optimization cost function includes multiple parameters. Consequently the problem of the output energy of the piezoelectric system is analyzed for the “worst case” scenario.

If (experimental) sample data is available, the OUQ approach can be extended to the Machine Wald^[34] technique, which is equivalent to performing Bayesian inference but optimizing the prior value. In Machine Wald, if an estimation of a function $\Phi(\mu)$, is function θ of sample data d , then the estimation error $\theta(d) - \Phi(\mu)$ is required to be equal to zero.

6 Conclusion

The unavoidable uncertainties in the material, loading, boundary conditions, and system efficiency of the complex bistable piezoelectric flutter energy harvesting model call for a probabilistic approach. In this paper, a realistic model of the bistable piezoelectric system which takes into account the uncertainties in the model with bounded constrained was pro-

posed, as a well-defined problem. This approach allowed obtaining a solution for the bounded output energy in accordance with the bounded inputs and system characteristics, using Optimal Uncertainty Quantification. The OUQ approach was recommended as a more reliable method in estimating the output energy of the system rather than relying merely on the deterministic approach. The future work of this research will involve comparing the experimental results with the numerical solutions of the corresponding OUQ-based model.

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