Adaptive stochastic resonance method for weak signal detection based on particle swarm optimization

XING Hongyan[1,2 *], ZHANG Qiang[1,2], LU Chunxia[1,2]

(1. *Collaborative Innovation Center on Forecast and Evaluation of Meteorological Disasters*, *Nanjing University of Information Science & Technology*, *Nanjing* 210044, *China*)

(2. *Key Laboratory of Meteorological Observation and Information Processing of Jiangsu Province*, *Nanjing University of Information Science & Technology*, *Nanjing* 210044, *China*)

Abstract: In order to solve the parameter adjustment problems of adaptive stochastic resonance system in the areas of weak signal detection, this article presents a new method to enhance the detection efficiency and availability in the system of two-dimensional Duffing based on particle swarm optimization. First, the influence of different parameters on the detection performance is analyzed respectively. The correlation between parameter adjustment and stochastic resonance effect is also discussed and converted to the problem of multi-parameter optimization. Second, the experiments including typical system and sea clutter data are conducted to verify the effect of the proposed method. Results show that the proposed method is highly effective to detect weak signal from chaotic background, and enhance the output SNR greatly.

Key words: Adaptive stochastic resonance; two-dimensional Duffing oscillator; weak signal detection; particle swarm optimization

1 Introduction

Chaos [1] is the irregular movement produced by nonlinear system, which widely exists in meteorology, hydrology, communications and other fields [2-4], such as sea clutter, electrical signals and Electrocardiograph (ECG). Sea clutter is the backscatter from sea surface and is usually treated as noise in traditional processing methods, which is easy to damage the useful signal. With the development of the wave's internal mechanisms, the chaotic and fractal features of sea clutter are found [5-6], which is influenced by wave, wind and other complex factors.

In recent years, the nonlinear science has made considerable progress. Many new methods have been put forward, and some researchers [7-9] have tried to use nonlinear system to detect weak signal. Stochastic resonance (SR) was proposed by Benzi [10] et al, when they studied the ancient glacier meteorological problems. Since then, SR has been broadly applied in the fields of signal processing [11-13] and has shown unique advantages to detect the weak signal in strong background noise. Due to the synergy of the input signal and noise, SR produces resonance output to strengthen the weak signal. In the field of chaos, Duffing oscillator [14] is a common nonlinear chaos system, and it is sensitive to the chaotic parameters and immune to noise, which requires the original frequency of Duffing system has to match the detected signal. Previous studies on SR mostly concentrated on Langevin system[15-17], in contrast, the discussion about two-dimensional Duffing oscillator is relatively rare and mainly focused on electronic circuit simulation [18-19] or theoretical level[20-21]. Therefore, the research on the synergistic effect of parameters is the key point to improve the usefulness of the theory of stochastic resonance for weak signal detection.

This article investigates the detection of weak signalunder the chaotic background, based on two-dimensional Duffing oscillator and particle swarm optimization (PSO), and analyzes the effects of different parameters on weak signal detection respectively. The method utilizes PSO to search the global optimal parameters to enhance the system of two-dimensional Duffing for detecting weak periodic signal contained in sea clutter. In addition, to evaluate its-detection capability, results are compared with other detection method.

The rest of this article is organized as follows, Section2 describes the fundamental theory of two-dimensional Duffing oscillator and preliminarily analyzes the influence of parameters. Section 3 describes the adaptive stochastic resonance based on PSO and gives the steps of weak signal detection. Section4 uses other methods to verify the performance of this new detection method. Section 5 gives a conclusion.

2 Detection features of Duffing oscillator

2.1 Two-dimensional Duffing oscillator

The signal $s(t)$ and noise $n(t)$ drive the oscillator equation:

$$\ddot{x} + k\dot{x} - ax + bx^3 = s(t) + n(t) \qquad (1)$$

Where $\ddot{x}$ and $\dot{x}$ are second and first order differential respectively, k is damping ratio, $-ax + bx^3$ is the nonlinear restoring force, a and b are system parameters which are both greater than zero. Assuming $s(t) = A\cos(2\pi f_0 t)$ is the harmonic signal where A is amplitude and f_0 is frequency. Besides, $n(t) = \sqrt{2D}\,\xi(t)$ is the white Gaussian noise, where D is noise intensity, $\xi(t)$ is Gaussian noise whose average is 0 and variance is 1. Therefore, equation (1) indicates the two-dimensional Duffing oscillator is driven by harmonic and noise.

$$\ddot{x} + k\dot{x} - ax + bx^3 = A\cos(2\pi f_0 t) + \sqrt{2D}\,\xi(t) \qquad (2)$$

When there is no additional signal, $s(t) = n(t) = 0$. The potential function $U(x) = -\frac{a}{2}x^2 + \frac{b}{4}x^4$ has two minimums ($x = \pm\sqrt{a/b}$) and a maximum ($x = 0$) in the middle, which causes symmetrical barrier and two potential wells. The result is shown in Figure 1, which means the Duffing system is a bistable system. When signal $A\cos(2\pi f_0 t)$ enters, the system has a critical value A_C ($\sqrt{4a^3/27b}$) theoretically. In fact, signal and noise could achieve synergies if $A < A_C$, therefore it would transfer some energy of noise to the signal and the system generates stochastic resonance. As for weak signal detection, the goal is to shift the energy of noise to the weak signal as much as possible.

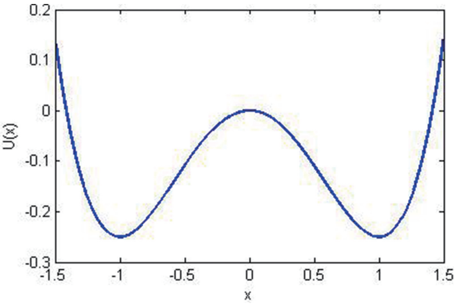

Fig. 1 Potential function of Duffing system (a=b=1)

2.2 The influence of parameters

According to equation (1), there are four parameters (i.e., k, a, b, and A) likely to influence the performance of the system. So that, we set a typical equation to analyze and discuss the differences, where $k = 0.5$, $a = b = 1$, $A = 0.1$, $f_0 = 0.01\text{Hz}$, $D = 0.4$, the sampling frequency $f_s = 5\text{Hz}$. Equation (2) uses the fourth order Runge-Kutta method to get the numerical solutions. However, the weak periodic signals are hidden in the noise and the frequency peak is not obvious, which is shown in Figure 2. When $D = 0.4$, the system achieves stochastic resonance.

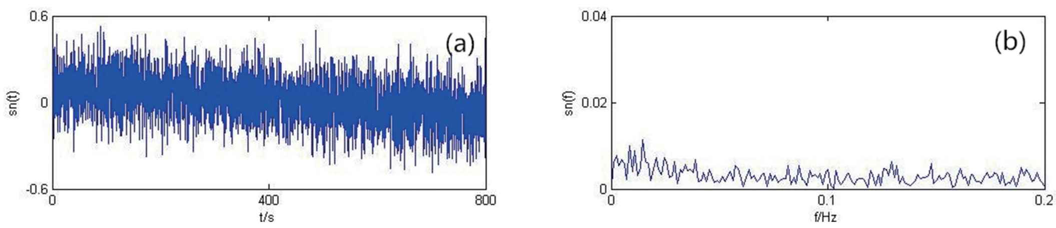

Fig. 2 Input signal (a) Waveform of input signal; (b) Frequency spectrum of the input signal

In addition, the frequency spectrum of the output is shown in Figure 3(c), which attains its maximum at f_0 and larger than the spectrum peak of input.

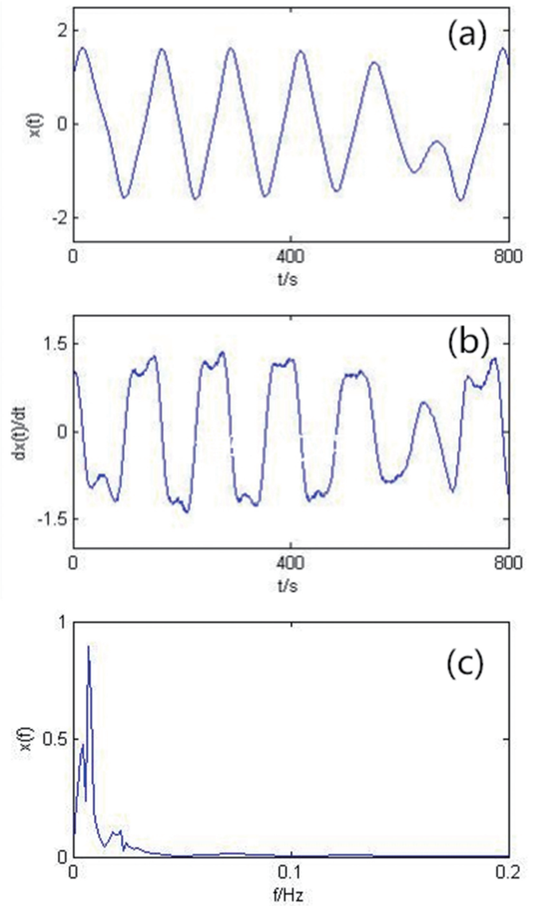

Fig. 3 Output signal
(a) Waveform of output signal; (b) Waveform of $dx(t)/d(t)$; (c) Frequency spectrum of the output signal

It is worth noting that there might exist better results than Figure 3(c). Hence, to find the optimum setting, the values of noise intensity, damping ratio and system parameters are changed respectively, and other parameters are kept constant at the same time. Figure 4 shows the frequency spectrum peak of the characteristic signal changes significant with the changes of system parameters. Obviously, with the increase of the parameter value, the curves of k, a, b are all rises then falls except D, which means there is a set of values to strengthen the energy of weak periodic signal.

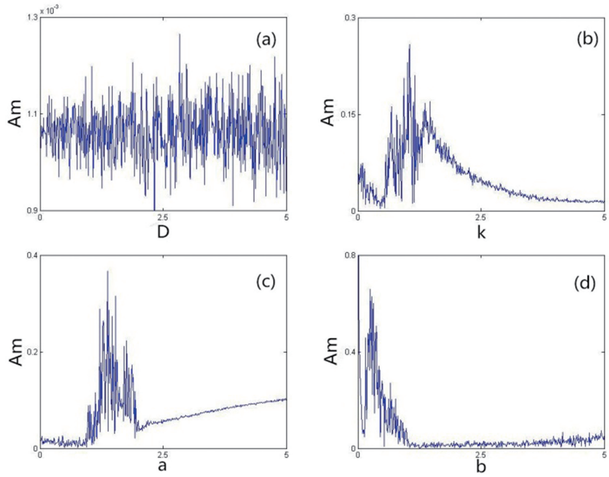

Fig. 4 The influence of parameters
(a) Noise intensity; (b) Damping ratio;
(c) System parameter a; (d) System parameter b

For the purpose of enhancing the practicability of SR, literature[22] presents a method about transforming the amplitude and scale of equation (1) for the detection of weak signals with arbitrary frequency and amplitude:

$$\ddot{x} + k\dot{x} = \lambda\left[ax - bx^3 + A\cos\left(2\pi\frac{f_0}{R}t'\right) + \sqrt{2D}\xi(t')\right] \quad (3)$$

Where λ is the amplitude transform coefficient, R is the scale coefficient. If the original f_0 doesn't satisfy the small parameter condition, the f_0/R can adjust to meet the requirements, also $t' = Rt$. What's more, these added parameters are just for the sake of practical need of engineering, while won't affect the features of stochastic resonance.

3 Adaptive stochastic resonance based on PSO

Generalization ability is the key indicator for stochastic resonance, which is related to the damping ratio and system parameters. At present, climbing method [23] and genetic algorithm [24] are used commonly. The prior one has high precision, but is easy to be trapped in local minima. Genetic algorithm (GA) is a kind of evolutionary algorithm using the selection, crossover and mutation to find the optimal value, however the codec is complicated and the preferences mostly rely on experience. In 1995, Particle swarm optimization [25] (PSO) was proposed by Eberhart et al, and this method is simple and easy to achieve the overall optimal state. PSO abandons the steps of crossover and mutation; meanwhile, it could find the global optimal value by following the current search, which has the merits of high accuracy, fast convergence and good operability.

3.1 Optimization process

First of all, the particle swarm is initialized to a group of random particles and finds the optimal solution by iterating calculation. Then particles update themselves by tracking two extreme values by iteration, which contains the particle itself and the entire population. Assuming particles form a community of the M-dimensional space, where i_{th} particle is expressed as a M-dimensional vector:

$$X_i = (x_{i1}, x_{i2}, \cdots, x_{iM}),\ i = 1,2,\cdots,N \tag{4}$$

Also, the "flight" speed of i_{th} particle is a M-dimensional vector:

$$V_i = (v_{i1}, v_{i2}, \cdots, v_{iM}),\ i = 1,2,\cdots,N \tag{5}$$

The best locationthe i_{th} particle has searched is called the individual extreme:

$$p_{best} = (p_{i1}, p_{i2}, \cdots, p_{iM}),\ i = 1,2,\cdots,N \tag{6}$$

Meanwhile, the optimal location of the whole particle swarm is the global extreme:

$$g_{best} = (p_{g1}, p_{g2}, \cdots, p_{gM}),\ i = 1,2,\cdots,N \tag{7}$$

When the algorithm finds the two optimal values, the particles update the speed and position by equation (8) and (9):

$$v_{id} = w \times v_{id} + c_1 r_1 (p_{id} - x_{id}) + c_2 r_2 (p_{gd} - x_{id}) \tag{8}$$

$$x_{id} = x_{id} + v_{id} \tag{9}$$

Where c_1 and c_2 are learning factors, r_1 and r_2 are uniform random numbers within the scope of $[0,1]$. Usually, $c_1 = c_2 = 2$, $i = 1,2,\cdots,M$, but the studies of Parsopoulos and Vrahatis[26] have shown that $c_1 = c_2 = 0.5$ is better. v_{id} is the speed of the particles, $v_{id} \in [-v_{\max}, v_{\max}]$.

3.2 The steps of adaptive stochastic resonance based on PSO

According to Section 2, even the same input signal, the different values of the system parameters have effects on the results in varying degrees. Therefore, the problem of adaptive stochastic resonance is converted into multi-dimensional continuous optimization. Furthermore, the SNR of input and output signal can directly reflect the performance of weak signal detection, also be taken as the fitness function of PSO:

$$\text{fitness}(x) = SNR_{out} = 10\lg \frac{S}{N}(dB) \tag{10}$$

Where $S = 2\,|x(f_0)|^2$, $N = \sum_{f=0}^{L-1} |x(f_0)|^2 - S$, $x(f_0)$ is the discrete Fourier transform of the sampling sequence. The process is shown in Figure 5, and the main steps are as follows:

Step 1 Initialize the particle swarm. Set the biggest iteration steps T, population P, dimension D, optimum range R in each dimension. Besides, initialize the position vector of the particles randomly.

Step 2 In addition, initialize the best fitness value and calculate the fitness values of each particle based on the Equation (10). Moreover, choose the first generation of the fitness value $p_{best}(i)(i = 1, 2,\ldots,N)$ as the local optimal fitness value, and the biggest value of g_{best} is called the global optimal fitness value.

Step 3 Update the best fitness value. Furthermore, update speed and position constantly according to the particle swarm fitness and enhance the system until the optimal state.

Step 4 Choose the particle location as the optimal parameters according to the final values of the global optimal fitness. So far, the optimal adaptive stochastic resonance is established and can be applied for weak signal detection. In addition, analyze the frequency spectrum of the output and calculate the SNR including the input and output of the signal.

4 Experiment and discussion

Experiment 1: In order to verify the proposed method, the periodic signal is firstly used to simulate. In Equation (1), the damping ratio k is largely

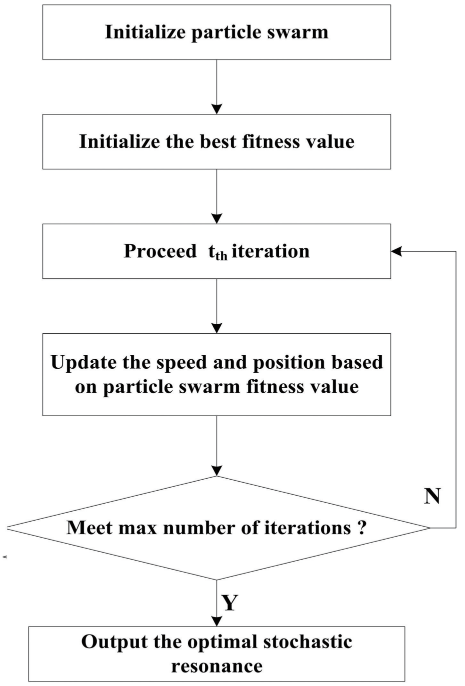

Fig. 5 The steps of adaptive stochastic resonance based on PSO

to determine the free vibration attenuation speed and the energy dissipation rate of the stochastic resonance. While the particles want to match the periodic change of the potential function, system needs stronger noise to provide enough energy. Therefore, the selection of damping ratio k also needs some ways. However, there are three parameters (k , a , b) in the system of two-dimensional Duffing, which makes it more difficult to adjust parameters.

In addition, this article firstly selects controlling variable method to validate the performance of the proposed method under different conditions. While it has to ignore the interaction between different parameters, but the deficiency can be made up by the features of PSO. Therefore, we respectively select the weak signal amplitude $A = 0.10:-0.01:0.01$, noise intensity $D = 0.1:0.1:1.0$, and get 10 sets of original signal. The optimizing results are shown in Table 1, the Δ_{SNR} has been greatly enhanced along with the rise in D. On the other hand, although the SNR of the input continues to decline, but the SNR of the output remains relatively constant, which means the performance of the proposed method is stable and can be completely applied to the condition of strong noise.

Experiment 2: As to sea clutter, it has been proved that can be dealt with as colored noise, affected by factors like wind and wave; also its power spectral density changes with the reciprocal of frequency. Above all, these features will distinctly increase the difficulty of detecting the weak signals submerged in sea clutter. Therefore, Experiment 2 adopts the sea clutter that contains small target to verify the performance of the proposed method. The data is measured by Canadian McMaster IPIX radar[27], and its emission frequency is 9.39 GHz, pulse repetition frequency is 1000Hz. Besides all above, each

Table 1 The optimizing results

Serial number	A	D	k	a	b	SNR_{in}/dB	SNR_{out}/dB	Δ_{SNR}/dB
1	0.10	0.1	0.0952	0.1678	0.2224	-1.8079	27.4364	29.2443
2	0.09	0.2	0.1188	0.3815	0.2238	-5.1188	27.4709	32.5897
3	0.08	0.3	0.0741	0.1177	0.3382	-7.8476	27.0004	34.8480
4	0.07	0.4	0.1266	0.1567	0.1318	-10.3314	27.4067	37.7381
5	0.06	0.5	0.1439	0.1271	0.0937	-12.3118	27.4307	39.7425
6	0.05	0.6	0.1302	0.1648	0.4827	-13.3991	27.2052	40.6043
7	0.04	0.7	0.0891	0.1669	0.1911	-15.7862	27.3142	43.1004
8	0.03	0.8	0.2308	0.1419	0.3656	-19.8074	28.6813	48.4887
9	0.02	0.9	0.1053	0.1714	0.3712	-22.9575	27.4524	50.4099
10	0.01	1.0	0.0864	0.1478	0.2474	-23.9029	30.0117	53.9146

set of data contains 131072 sampling points and uses VV polarization mode. What´s more, because the data contains a periodic signal, and the characteristic of sea clutter is similar to the noise, which means it can rightly replace the right side of Equation (1).

For the purpose of satisfying the requirement of stochastic resonance, the experiment firstly adopts the Equation (3) to change the frequency and amplitude. Then using the same steps of Experiment 1 to output the result. Finally, the frequency spectrum is analyzed to validate whether there exist the weak periodic signal, meanwhile calculates the final parameters and SNR.

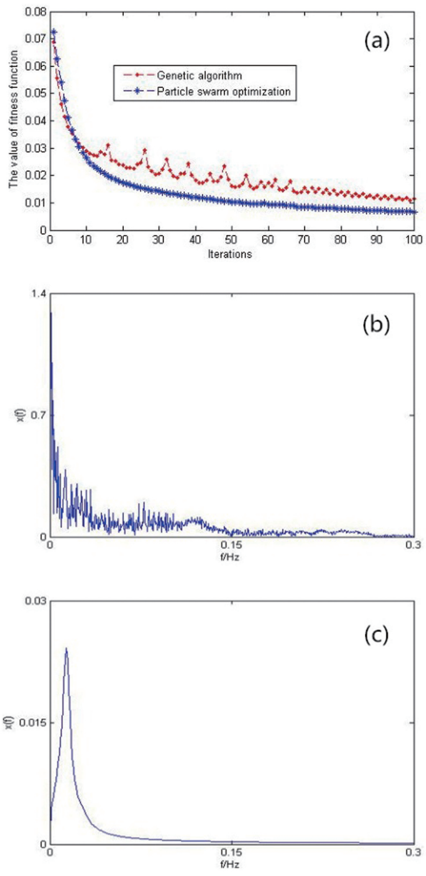

Fig. 6 The optimization results

(a) The iterative process; (b) Frequency spectrum of sea clutter; (c) Frequency spectrum of output signal

In order toverify the effect of the proposed method, we choose the Genetic Algorithm as the comparison object. Figure 6(a) shows PSO can get the optimal parameters with limited steps, and the performance of it is much better than genetic algorithm especially in the case of local optimum. The optimized system ultimately gets $k = 0.8367$, $a = 1.1104$, $b = 0.8367$, $SNR_{in} = -12.0434$ and $SNR_{out} = 32.4820$dB. In addition, the results are even better than Experiment 1 and literature[28] at the same level of SNR_{in}. Figure 6(c) shows there existing a peak frequency in 0.0137, which proves the energy of sea clutter can be transferred to the weak signal by the optimized system of two-dimensional Duffing. In order to avoid the contingency, many experiments are conducted later, and the results are entirely consistent. All the results made in the optimized system of two-dimensional Duffing prove that the method in the article to be effective further, so it is with its universality, what is flexible to practical problems.

5 Conclusion

This article utilizes the optimizing advantage of PSO for searching the global optimal parameters, then the optimized results for stochastic resonance is applied to utilize the system of two-dimensional Duffing to detect weak signal. As for system parameters, we put focus on the features when each parameter changes respectively, especially the change of SNR_{out}. When the values of three parameters (i.e., k, a and b) increases, the trends are consistent and exist the maximum of each change, which means there will be a set of optimal values can transform the energy of noise into signal as well as enhanceSNR_{out}.In addition, we select different amplitude and noise intensity as test data for verifying the proposed method. With the increase of noise intensity, SNR_{out} basically unchanged, while Δ_{SNR} has a significant rise, which means the optimized adaptive stochastic resonance is suitable for strong noise environment better. Moreover, sea clutter is added for testing the practicability of the proposed method and the result is even better than the simulation, so that the accuracy, robustness and generalization are advanced.

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Authors' Biographies

XING Hongyan, born in 1963, is currently a Professor in the school of Electronics and Information Engineering, Nanjing University of Information Science and Technology, Nanjing, China. He obtained the Ph.D. degree in Biomedical engineering from Xi´an Jiaotong University, Xi´an, China in 2003. His current interests concentrate on design and metering of meteorological instruments, and signal detection and processing.

E-email: xinghy@nuist.edu.cn

ZHANG Qiang, born in 1990, is currently a graduate student in the school of Electronic and Information Engineering, Nanjing University of Information Science and Technology, Nanjing, China. She obtained her undergraduate degree from Nanjing University of Information Science and Technology in 2013. His current research interests include stochastic resonance, data mining and weak signal detection method.

LU Chunxia, born in 1992, is currently a graduate student in the school of Electronic and Information Engineering, Nanjing University of Information Science and Technology, Nanjing, China. She obtained her undergraduate degree from Nanjing University of Information Science and Technology in 2014. Her current research interests include stochastic resonance, neural network, and weak signal detection method.