Frequency modulated weak signal detection based on stochastic resonance and genetic algorithm

XING Hongyan[1,2*], LU Chunxia[1,2], ZHANG Qiang[1,2]

(*1. Collaborative Innovation Center on Forecast and Evaluation of Meteorological Disasters, Nanjing University of Information Science & Technology, Nanjing* 210044, *China*

2. Key Laboratory of Meteorological Observation and Information Processing of Jiangsu Province, Nanjing University of Information Science & Technology, Nanjing 210044, *China*)

Abstract: Stochastic resonance system is subject to the restriction of small frequency parameter in weak signal detection, in order to solve this problem, a frequency modulated weak signal detection method based on stochastic resonance and genetic algorithm is presented in this paper. The frequency limit of stochastic resonance is eliminated by introducing carrier signal, which is multiplied with the measured signal to be injected in the stochastic resonance system, meanwhile, using genetic algorithm to optimize the carrier signal frequency, which determine the generated difference-frequency signal in the low frequency range, so as to achieve the stochastic resonance weak signal detection. Results show that the proposed method is feasible and effective, which can significantly improve the output SNR of stochastic resonance, in addition, the system has the better self-adaptability, according to the operation result and output phenomenon, the unknown frequency of the signal to be measured can be obtained, so as to realize the weak signal detection of arbitrary frequency.

Keywords: stochastic resonance; two-dimension Duffing oscillator; frequency modulated; genetic algorithm.

1 Introduction

Stochastic resonance (SR) is proposed by Benzi[1-2] et al when they studied the ancient glacier meteorological problems in 1981, which describes the phenomenon that the noise energy is converted into signal energy when the input signal , noise and the nonlinear system achieve synergy effect, so as to extract the weak signal from background noise. The noise can be regarded as active element in the stochastic resonance system, which is different from the traditional method of removing noise[3-6]. The unique advantages of stochastic resonance aroused wide attention of domestic and foreign scholars, it has been successfully applied to different fields [7-14].

The two-dimensional Duffing stochastic resonance system is a nonlinear system, which can produce chaos phenomena, using its sensitivity to initial value and immunity to noise, we can achieve the weak signal detection under strong noise background[15-16], but the method is limited by the small frequency parameters, can only detect low frequency signal. For the high frequency signal, the traditional stochastic resonance signal detection method is scale transform[17], which makes the driving signal frequency of Duffing stochastic resonance system be matched with the measured signal frequency, although this method extends the parameter range of the stochastic resonance weak signal detection system, it requires that the parameter matching of the scale transformation should have a certain precision, which is difficult to operate. Therefore, combined with genetic algorithm[18], which has good global search ability, a frequency modulated weak signal detection method based on stochastic resonance and genetic algorithm is proposed, this method can avoid the matching parameters selection problem of the nonlinear system, and can effectively detect the high frequency signal.

This paper derives the principle of frequency modulation, based on the stochastic resonance characteristics of Duffing oscillator [19], it uses two-dimensional Duffing oscillator as the taking carrier of stochastic resonance, introducing the carrier signal, which is multiplied with the signal to be measured in the mixer, at the same time, using genetic algorithm to find the optimal carrier signal frequency, and the generated offset signal is directly loaded into the Duffing oscillator model to be the driving signal of the stochastic resonance bistable system, which construct a weak signal detection method of arbitrary large frequency.

2 Theoretical analysis

2.1 Stochastic resonance theory based on two-dimension Duffing oscillator

Duffing oscillator differential equation driven by signal and noise, whose general form is:

$$\ddot{x} + \delta\dot{x} - ax + bx^3 = s(t) + n(t) \qquad (1)$$

Equation (1) can also be written as the form of the system:

$$\begin{cases} \dot{x} = v \\ \dot{v} = ax - bx^3 - \delta v + s(t) + n(t) \end{cases} \qquad (2)$$

Where δ is damping ratio, $-ax + bx^3$ is the nonlinear restoring force, a and b are system parameters that are greater than zero, $s(t) = A\sin(2\pi f_0 t)$ is the weak periodic signal to be measured, where A is driving force amplitude and f_0 is frequency, $n(t) = \sqrt{2D}\xi(t)$ is the white Gaussian noise where D is noise intensity, the research shows that when the nonlinear system parameters $a = b = 1$, $\delta = 0.5$, the system which meet the conditions of small frequency signal has good synergistic effect. When the Duffing system without any input signal ($s(t) + n(t) = 0$), the potential function is $U(x) = -\frac{a}{2}x^2 + \frac{b}{4}x^4$, which describes a bistable system with two minimum values ($x = \pm\sqrt{a/b}$) and a maximum value ($x = 0$), whose barrier height is $\Delta U = a^2/4b$, the result is shown in Fig 1. When the signal $s(t)$ is entered, the system gets a threshold $A_c = \sqrt{4a^3/27b}$. Only when $A > A_c$, at the same time, the nonlinear system and excitation signal achieve synergies, the noise energy will transfer to signal energy, and the system output will jump between two steady states in a large range, that is to say, the system enters the stochastic resonance state.

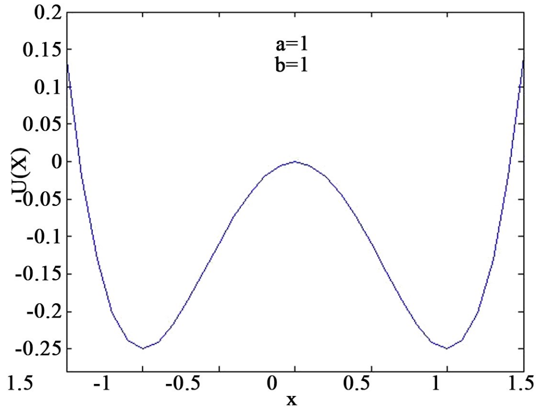

Fig. 1 Potential function of Duffing system (a = b = 1)

The adiabatic approximation theory of stochastic resonance[20] makes the stochastic resonance system only suitable for small parameter conditions, Fig. 2 is a typical small parameter stochastic resonance processes, stochastic resonance system parameters in the Equation (1): $\delta = 0.5$, $a = b = 1$, the input signal and noise parameters: $A = 0.1$, $f_0 = 0.01\text{Hz}$, $D = 0.4$, the sampling frequency is $f_s = 5\text{Hz}$. The Runge-Kutta algorithm is used to solve the two-dimensional

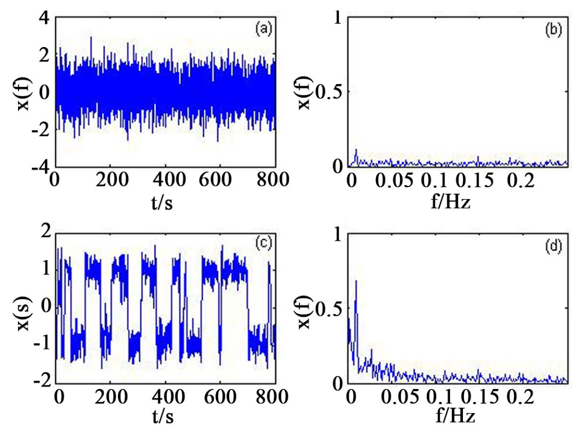

Fig. 2 Typical stochastic resonance phenomenon
(a) input signal waveform; (b) input signal spectrum; (c) output signal waveform; (d) output signal spectrum;

Duffing oscillator equation, whose calculation points is 4096, and the output spectrum is averaged by 10 times. From Fig.2 we can see that the system produces resonance output when the small frequency parameter $f_0 = 0.01\text{Hz}$, where the spectral peak of the output spectrum reaches the maximum, and it is much larger than the peak of the input spectrum, here the output signal to noise ratio (SNR) is $SNR_{0ut} = 32.3284$.

2.2 The principle of frequency modulation

In the actual engineering, we encountered is mostly high frequency signal, in order to apply stochastic resonance to the detection of high frequency signals, this article introducing carrier signal $c(t) = \cos(2\pi f_c t)$ with variable frequency f_c (using genetic algorithm to determine the value of f_c), which can modulate the weak periodic signal to be measured, in order to be conducive to the system output, the carrier signal amplitude take 1. The principle diagram of the frequency modulation signal detection is shown in Fig.3.

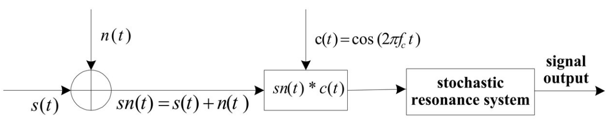

Fig. 3 The principle diagram of the frequency modulation signal detection

Mixed background signal $sn(t) = s(t) + n(t)$ input the system, which is multiplied by the carrier signal, so that produce the offset signal (difference-frequency and sum-frequency signal). Such as Formula (3):

$$sn(t) * c(t) = [A\cos(2\pi f_0 t) + n(t)] * \cos(2\pi f_c t) = \frac{A}{2}\cos(2\pi(f_c - f_0)t) + \frac{A}{2}\cos(2\pi(f_c + f_0)t) + n(t) * \cos(2\pi f_c t) \tag{3}$$

As shown in Formula(3), the original input signal amplitude is A, and the original carrier signal amplitude is 1, after mixing, the new signal amplitude is reduced by half, however, the noise component is also reduced, so the input SNR of excitation signal is substantially the same, and the nature of stochastic resonance will not be changed. When the nonlinear bistable system is affected by noise and the signal containing high and low frequency, due to the adiabatic approximation theory of stochastic resonance, the system is more sensitive to the difference-frequency (lower frequency) component $(f_c - f_0)$, and the sum-frequency component $(f_c + f_0)$ of Formula (3) through stochastic resonance bistable system basically disappeared. In the frequency spectrum of the output signal, the maximum value of spectral peaks will appear at the frequency $f = f_c - f_0$, according to the output phenomenon and the carrier signal frequency, we can accurately measure the frequency f_0 .

In the case of over damping, the inertia term of the two-dimensional Duffing oscillator equation can be ignored, which can obtained the one-dimensional Langevin equation:

$$\delta\ddot{x} - ax + bx^3 = s(t) + n(t) = A\cos(2\pi f_0 t) + \sqrt{2D}\xi(t) \tag{4}$$

Or:

$$\dot{x} - a'x + b'x^3 = A'\cos(2\pi f_0 t) + \sqrt{2D'}\xi(t) \tag{5}$$

Where:

$$\begin{cases} a' = a/\delta \\ b' = b/\delta \\ A' = A/\delta \\ D' = D/\delta^2 \end{cases} \tag{6}$$

According to the Formula (6) and (5), the damping ratio δ can be adjusted by changing the a' 、b' 、A' 、D' , therefore, the two-dimensional Duffing equation can be understood as a one-dimensional Langevin equation with variable damping ratio. Based on the one-dimensional stochastic resonance kramers escape rate theory[21], the transition rate of particles in the two potential well is:

$$r_k = \frac{a'}{\sqrt{2\pi}}\exp(-\frac{\Delta U'}{D'}) \tag{7}$$

The Formula (6) and $\Delta U' = a'^2/4b'$ into the

Formula (7):

$$r_k = \frac{a}{\sqrt{2}\pi\delta}\exp(-\frac{a^2\delta}{4bD}) \tag{8}$$

Assume that the general two-dimensional Duffing nonlinear stochastic resonance system parameters $a = b = 1$, damping ratio $\delta = 0.5$, when the noise intensity D is very large, according to the Formula (8), we calculate the theoretical limit value $r_{k\max} \approx 0.450\text{Hz}$, therefore, the two-dimensional stochastic resonance system can only produce resonance with the low frequency ($0 < f < 0.225\text{Hz}$) signal. For the high frequency weak signal, using the genetic algorithm monitoring the stochastic resonance output SNR to search for the carrier signal frequency f_c, by frequency modulation, a new low frequency signal can be obtained, which satisfy $0 < f < 0.225\text{Hz}$, and the matching relationship is achieved with stochastic resonance nonlinear system, so as to induce stochastic resonance.

3 Frequency modulated weak signal detection based on stochastic resonance and genetic algorithm

Generalization ability is an important indicator for the performance of stochastic resonance systems, which is related to the forecast accuracy of the system to induce stochastic resonance with low frequency signal. The ultimate goal of optimizing the stochastic resonance carrier signal frequency f_c is to enhance the system generalization ability, so as to improve the output SNR. Genetic algorithm is a highly parallel, random and adaptive global optimization method which is based on the biological evolution mechanism, through performing selection, crossover and mutation operation, the individual with high fitness as the optimal solution is finally evolved by several iterations. At present, utilizing the global optimization ability of the genetic algorithm to make the stochastic resonance system reach the optimal state.

3.1 Encoding and decode method

Encoding and decoding problem is about the mapping between real space and genetic space, the data in real space is obtained by encoding to get the gene in genetic space, and the gene in genetic space is decoded to get the data in the actual space. There is only one optimization parameter, so adopting the binary encoding method, which has the advantages of simple operation, easy implementation.

3.2 Fitness function

Genetic algorithm is based on the fitness function to find the optimal individual, and SNR is a key indicator to measure the effect of stochastic resonance. Therefore, choose the Duffing oscillator stochastic resonance output SNR as the fitness function (objective function) of genetic algorithm, defined as:

$$Y = 10\lg\frac{S(f_c - f_0)}{N(f_c - f_0)} \tag{9}$$

Where $S(f_c - f_0)$ is the amplitude of the signal power spectrum at the frequency of $f_c - f_0$, $N(f_c - f_0)$ is the average power of noise.

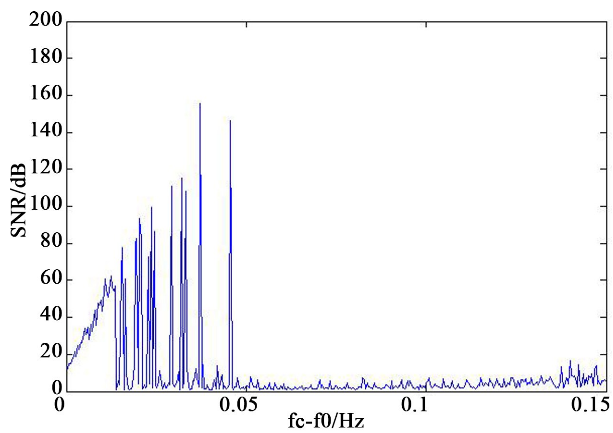

Fig. 4 The variation of the SNR with f_c-f_0

Fig.4 is the variation regularity of the SNR with the change of the difference-frequency signal frequency. After the signal is processed by the Duffing stochastic resonance frequency modulated system, the output SNR is increased substantially at low frequency, at the same time, there are a number of low frequency points, which can make the system reach stochastic resonance. The output SNR is different, the stochastic resonance effect is not the same, therefore, the output SNR is used as the fitness function, and the role of genetic algorithm is to find the carrier signal frequency of the SNR at the maximum.

3.3 Genetic operation

The genetic operation mainly includes three kinds of operation modes: selection, crossover and mutation operation.

The purpose of selection is to select an excellent individual from the current population, the greater the fitness function value is, the greater the probability that the individual is selected.

Crossover is the operation of the two parents exchange information with probability η_i , which can generate a new individual, and the new individual inherits the characteristics of the parents.

Mutation refers to select an individual from the population at random, and randomly alter a bit in the binary data with a certain probability Ψ_i . Generally speaking, the probability of mutation is very low, usually take 0.001-0.01.

3.4 The main process

The process of frequency modulated weak signal detection based on stochastic resonance and genetic algorithm is shown in Fig.5. The main steps are as follows:

Step 1. Set up the system initialization parameters, encoding to the optimized parameter and generating initial population $P_i(t), i = 1,2,\cdots N$, whose individuals is N.

Step 2. Calculating fitness value (stochastic resonance output SNR), and the fitness function Y is used to evaluate the $P_i(t)$.

Step 3. Perform selection operation. The probability of selection is determined according to the fitness value, such as the Formula (10):

$$\Phi_i = \frac{Y_i}{\sum_{i=1}^{N} Y_i} \quad i = 1,2,3\cdots,N \qquad (10)$$

Where, N is the number of individuals, Y_i is the i-th individual fitness. Individuals with large fitness values are inherited to the next generation.

Step 4. Perform crossover and mutation operations for the current population, which can generate the next generation of groups.

Step 5. Return to step 2 until the optimal condition is satisfied (the optimal condition is that the algorithm reaches the maximum number of iterations). According to the output result and stochastic resonance phenomenon, restoring the true frequency.

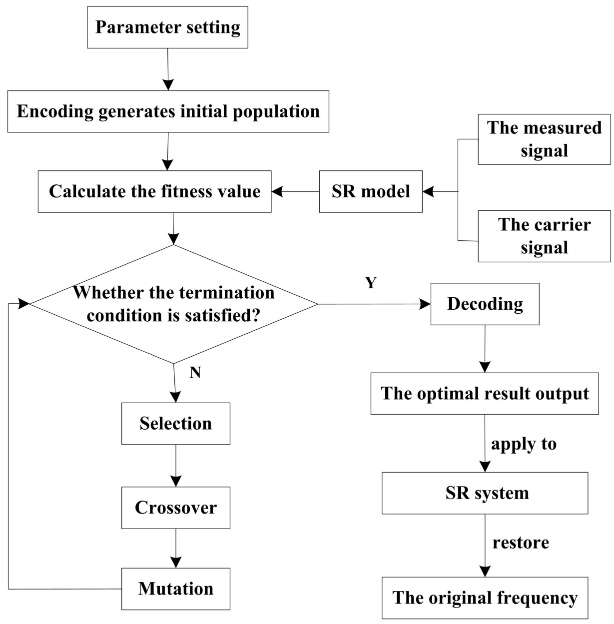

Fig. 5 The flow chart of the system

4 Experiment and discussion

According to the adiabatic approximation theory, the stochastic resonance weak signal detection is only effective for small parameter signals. In practical applications, the signal frequency parameters are often greater than 1, therefore, the classical stochastic resonance can not solve this problem. In order to verify the effectiveness of the proposed method, first, taking the high frequency signal as an example, and the simulation model as shown in Formula (2), it is assumed that the input high signal frequency $f_0 = 40\text{Hz}$, amplitude $A = 0.21$, and the noise intensity $D = 0.35$, carrier signal with frequency f_c is introduced into the system, and the generated low frequency (difference-frequency) is $f = f_c - f_0$. Duffing general stochastic resonance system parameters:

$a = 1$, $b = 1$, $\delta = 0.5$, sampling frequency $f_s = 5\text{Hz}$, sampling time $t = 500\ s$, the original equation is solved by adopting Runge-Kutta fourth-order algorithm, whose calculation points is 4096. The key parameters of genetic algorithm: population $N = 50$, crossover probability $\eta_i = 0.7$, mutation probability $\Psi_i = 0.005$, the number of iterations: $T = 100$, the accuracy of fitness is 0.001. Simulation results are shown in Fig. 6 and Fig. 7.

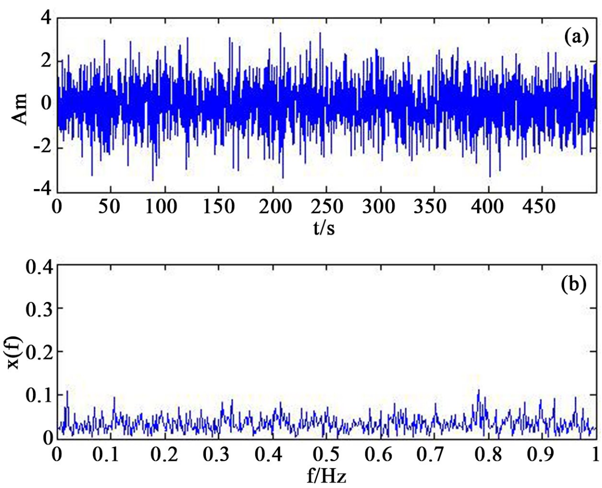

Fig. 6 Input signal
(a) input signal waveform (b) input signal spectrum

As shown in Fig.6 (a), Weak periodic components are submerged in strong noise background, even in the input spectrum of Fig.6 (b), the periodic component can not be found. After the high frequency signal is processed by the system, the system operation output is: $f_c = 40.02\text{Hz}$, $f = 0.02\text{Hz}$, $SNR_{out} = 53.3584$ dB, the corresponding resonance output phenomenon is shown in Fig.7 (c), its periodic component is obvious, and its output spectrum is shown in Fig.7 (d), the spectral peak at 0.02Hz is also prominent, this phenomenon is consistent with the result of the system output $f = 0.02\text{Hz}$. The original high frequency signal can be restored by $f_0 = f_c - f$, whose periodic component frequency is 40Hz.

In order to further verify the generality of the proposed method, using five different input signal data, and other things being equal, the optimizing results are shown in Table 1.

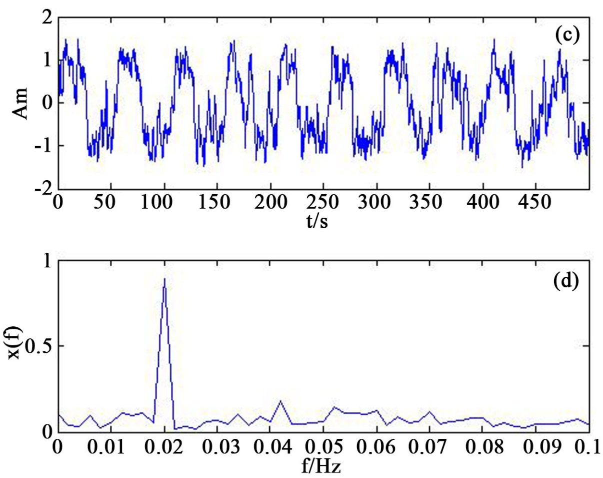

Fig. 7 Output signal ($f_c = 40.02Hz$)
(c) output signal waveform (d) output signal spectrum

Table 1 The optimizing results

Number	Input signal	D	D	i_{th}	D	$X_i = (x_{i1}, x_{i2}, \cdots, x_{iD})$
1	A=0.21, D=0.35	40	40.020	0.020	-13.6251	53.3584
2	A=0.08, D=0.27	30	30.051	0.051	-7.6398	56. 9582
3	A=0.06, D=0.18	50	50.030	0.030	-7.3974	49.5180
4	A=0.10, D=0.40	0.01	0.042	0.032	-10.0557	49.7381
5	A=0.05, D=0.22	0.02	0.043	0.023	-9.3602	46.3677

Simulation results show that the proposed method is efficient and feasible, which can greatly improve the output SNR (average increase of 60.80378dB). It can be seen from Table 1 that the frequency modulated weak signal detection based on stochastic resonance and genetic algorithm is not only suitable for the detection of high frequency signals, but also for the detection of low frequency signals. The signal in Fig.1 is directly loaded into the stochastic resonance system with a frequency of 0.01, whose output SNR is 32.3284 dB, under the same condition, in Table 1, serial number 4, the low frequency signal is processed by the proposed system, the output carrier signal frequency is f_c = 0.042Hz , and the stochastic resonance output phenomenon as shown in Fig.8, (Utilizing the proposed method to detect the low frequency signal, the generated difference-frequency and sum-frequency signal are both in the low frequency region, so, there are two spectrum peaks in the output spectrum). The system output spectrum reaches the maximum at the difference-frequency f = 0.032Hz , and the sum-frequency ($f_c + f_0$) signal appear at 0.052Hz, therefore, it is known that the original periodic signal frequency is 0. 01Hz, and the output SNR is 49.7381dB, which is higher than the traditional method.

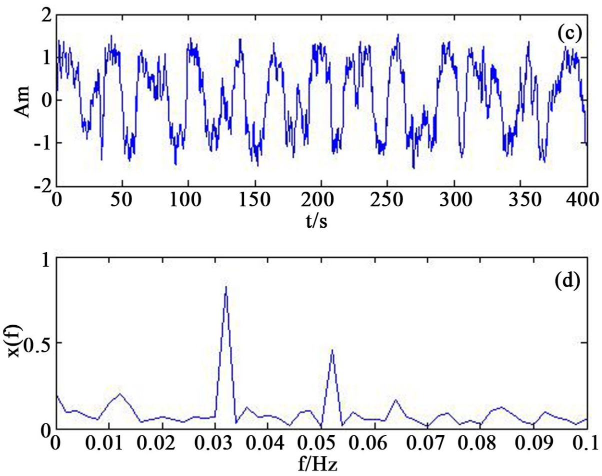

Fig. 8 Output signal (f_c = 0.042Hz)
(c) output signal waveform (d) output signal spectrum

5 Conclusion

Two-dimensional Duffing stochastic resonance is the optimal matching result between the signal, noise and nonlinear system, based on the stochastic resonance theory, this paper presents a method of frequency modulated weak signal detection, combined with genetic algorithm, where a new low frequency signal is produced, which is matched with the stochastic resonance system, so as to realize the stochastic resonance weak signal detection of arbitrary frequency. Numerical analysis and simulation results show that: this method can greatly improve the output SNR, compared with the traditional method, the proposed method has better generalization ability and higher accuracy of signal detection, to a certain extent, it improves the output performance of two-dimensional Duffing oscillator stochastic resonance, not only suitable for high frequency signal detection under strong noise background, but also suitable for low frequency weak signal detection, which can extended the stochastic resonance weak signal detection scope, and provide a theoretical support for Duffing oscillator stochastic resonance weak signal detection in actual engineering.

Acknowledgment

This work was supported by the National Natural Science Foundation of China (Grant No. 61072133), the Production, Learning and Research Joint Innovation Program of Jiangsu Province, China (Grant Nos. BY2013007-02, SBY201120033), the Industrialization of Research Findings Promotion Program of Institution of Higher Education of Jiangsu Province, China (Grant No. JHB2011-15), the advantage discipline platform "information and Communication Engineering" of Jiangsu Province, China, and the "Summit of the Six Top Talents" Program of Jiangsu Province, China.

References

[1] Benzi R, Sutera A, Vulpiani A. The mechanism of stochastic resonance[J]. Journal of Physics A Mathematical & General, 1981, 14(11):L453-L457.

[2] Benzi R, Sutera A, Vulpiani A. Stochastic resonance in the Landau-Ginzburg equation[J]. Journal of Physics A General Physics, 1998, 18(12):2239-2245.

[3] NIU J, SU T, HE X, et al. Weak NQR signal detection based on generalized matched filter[J]. Procedia Engineering, 2010, 7(8):377-382.

[4] LI J, LI H, LEI Z. Research on the Weak Signal Detection Based on Adaptive Filtering of Wavelet Transform[J]. Procedia Engineering, 2011, 15(1):2583-2587.

[5] YIN M, LIU W. Quaternion wavelet image denoising based on non-Gaussian distribution[J]. Journal of Electronic Measurement and Instrument, 2012, 26(4): 338-343.

[6] LU Y X, GU X H. Filtering algorithm for multi-microphones based on wavelet multi-scale information fusion[J]. Chinese Journal of Scientific Instrument, 2012, 33(4): 788-794.

[7] HE Q, WANG J. Effects of multiscale noise tuning on stochastic resonance for weak signal detection[J]. Digital Signal Processing, 2012, 22(4):614-621.

[8] ZHANG L. Stochastic resonance in a single-mode laser driven by quadratic pump noise and amplitude-modulated signal[J]. Chinese Physics B, 2009, 18(4):1389-1393.

[9] MIN G, FENG G. Phenomenon of Stochastic Resonance in a Time-Delayed Bistable System Driven by Colored Noise and Square-Wave Signal[J]. Chinese Journal of Physics- Taipei-, 2011, 49(2):655-663.

[10] ZHENG C H, WANG H, WANG H T. Entropic noise induced stability and double entropic stochastic resonance induced by correlated noises[J]. Chinese Physics B, 2011, 20(5):146-152.

[11] Li J H. Stochastic giant resonance.[J]. Physical Review E Statistical Nonlinear & Soft Matter Physics, 2007, 76(1):62-85.

[12] ZHAO W L, LIU J, YIN Y P. Medium-low frequency signal detection and circuit design based on stochastic resonance principle[J]. Chinese Journal of Scientific Instrument, 2011, 32(4): 721-728.

[13] XIA J Z, LIU Y H, MA Z P, et al. Weak signal detectionbased on the modulated stochastic resonance[J]. Journal of Vibration and Shock, 2012, 31(3): 132-135.

[14] LENG Y G, WANG T Y, GUO Y, et al. Study of the property of the parameters of bistable stochastic resonance[J]. Acta Physica Sinica, 2007, 56(1): 30-35.

[15] Novak S, Frehlich R G. Transition to chaos in the Duffing oscillator[J]. Phys. rev. a, 2011, 26(6):3660-3663.

[16] WANG N, ZHENG D Z. New method for speech endpoint detection under low SNR based on chaotic oscillator[J]. Chinese Journal of Scientific Instrument, 2009, 30(7): 1432-1435.

[17] DENG H, LENG Y G, WANG T Y. Study on the scale transformation concatenation stochastic resonance systembased on XPE[J]. Chinese Journal of Scientific Instrument, 2009, 30(10): 2033-2038.

[18] SHI G, JING Y W, XU A D, et al. Using an improved genetic algorithm to solve global optimization problem[J]. Chinese Journal of Scientific Instrument, 2006, 27(S3): 2329-2332.

[19] LENG Y G, LAI Z H, FAN S B. Large parameter stochastic resonance of two-dimensional Duffing oscillator and its application on weak signal detection[J]. Acta Physica Sinica, 2012, 63(23): 71-80.

[20] McNamara B, Wiesenfeld K. Theory of stochastic resonance. Phy. Rev, A, 1989, 39(9): 4854-4869.

[21] Gammaitoni L, Hanggi P, Jung P. Stochastic resonance.Rev Mod Phys, 1998, 70(1): 223-287.

Authors' Biographies

XING Hongyan, born in 1963, is currently a Professor in the school of Electronics and Information Engineering, Nanjing University of Information Science and Technology, Nanjing, China. He obtained the Ph.D. degree in Biomedical engineering from Xi´an Jiaotong University, Xi´an, China in 2003. His current interests concentrate on design and metering of meteorological instruments, and signal detection and processing.

LU Chunxia, born in 1992, is currently a graduate student in the school of Electronic and Information Engineering, Nanjing University of Information Science and Technology, Nanjing, China. She obtained her undergraduate degree from Nanjing University of Information Science and Technology in 2014. Her current research inter-

ests include stochastic resonance, neural network, and weak signal detection method.

ZHANG Qiang, born in 1990, is currently a graduate student in the school of Electronic and Information Engineering, Nanjing University of Information Science and Technology, Nanjing, China. He obtained his undergraduate degree from Nanjing University of Information Science and Technology in 2013. His current research interests include stochastic resonance, data mining and weak signal detection method.